MODULE :- 02

Design for Impact & fatigue loads

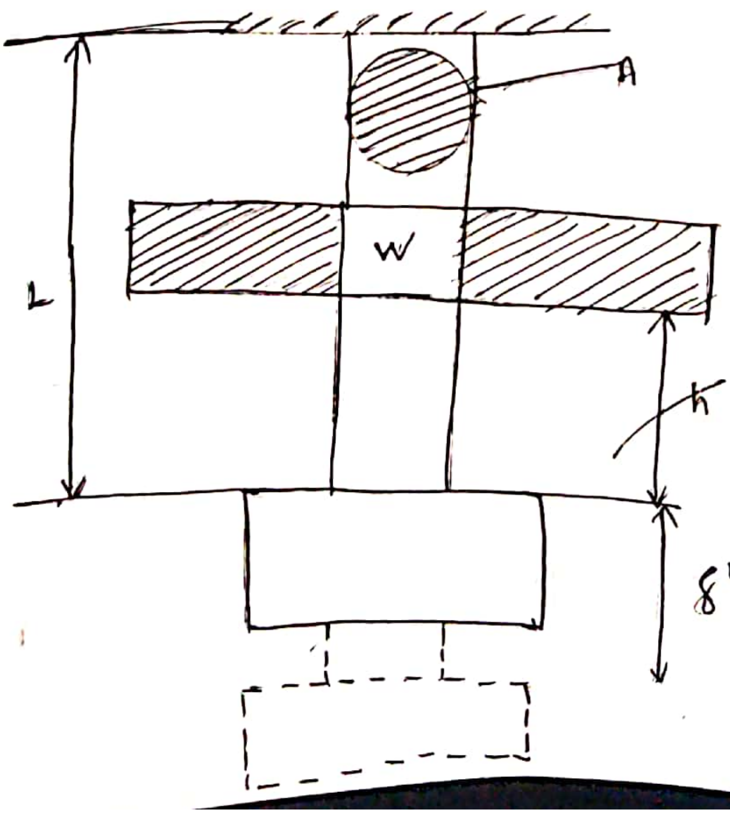
Impact stress :- Impact is defined as collision of ~~co~~ one component in motion with second component which ~~b~~ may be either in motion (or) rest.

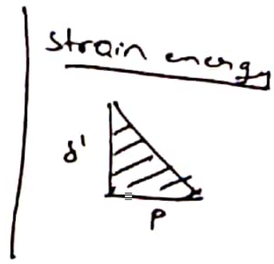
Impact load is the load which is rapidly apply to the machine component. The stress induced in the machine component. Due to the impact load is called Impact stress.

Impact forces are observed in machining component like Hammer, hoisting rope, clutches & Brakes, springs, punches etc.

~~Imp~~

Impact stress due to axial load on a bar





Consider an elastic body (or) system loaded by a falling weight W as shown in figure.

Let W = falling weight in N.

h = height through which weight fall in

L = Length of bar in mm.

δ' = Deformation due to impact.

A = Cross-sectional area of the bar.

E = young's modulus.

P = Impact force which produces deflection δ'

σ' = Impact stress in the bar.

δ = static deformation due to weight W

The weight W fall through the height 'h' & strikes the collar of the bar during this process the potential energy released by falling weight is absorb by the bar & stored in the form of strain energy.

Energy Released by falling weight 'W' = Potential energy

$$= W(h + \delta') \rightsquigarrow ①$$

~~Inter~~

Energy absorbed by collar = strain energy

= Average force × deflection

$$= \frac{P}{2} \times \delta' \rightsquigarrow ②$$

Equating ① & ②

$$w(h + \delta') = \frac{P}{2}\delta' \rightsquigarrow ③$$

wkT strain, $\varepsilon' = \frac{\delta'}{L}$

also, $E = \frac{\sigma'}{\varepsilon'}$ where E = youngs modulus

$$\varepsilon' = \frac{\sigma'}{E} = \frac{\delta'}{L}$$

$$\therefore \boxed{\delta' = \frac{\sigma' \times L}{E}}$$

stress, $\sigma' = \frac{P}{A}$

$$\boxed{P = \sigma' A}$$

substituting the values of P & δ' in eq^n ③

$$w\left(h + \frac{\sigma' L}{E}\right) = \left(\frac{\sigma' A}{2}\right)\left(\frac{\sigma' L}{E}\right)$$

$$wh + \frac{\sigma' L w}{E} = (\sigma')^2\left(\frac{AL}{2E}\right)$$

$$(\sigma')^2\left(\frac{AL}{2E}\right) - \sigma'\left(\frac{Lw}{E}\right) - wh = 0$$

wkT $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b = -\frac{WL}{E}$, $a = \frac{AL}{2E}$ & $c = -wh$

$$\sigma' = \frac{\left(\frac{WL}{E}\right) + \sqrt{\left(\frac{WL}{E}\right)^2 + \cancel{4}^{2}\left(\frac{AL}{\cancel{2}E}\right)(wh)}}{\cancel{2}\left(\frac{AL}{\cancel{2}E}\right)}$$

$$\sigma' = \frac{\frac{WL}{E} + \frac{WL}{E}\sqrt{1 + \frac{2AhE}{WL}}}{\frac{AL}{E}}$$

$$\sigma' = \cancel{\frac{L}{E}}\left[\frac{W + W\sqrt{1 + \frac{2AhE}{WL}}}{A \cdot \cancel{\frac{L}{E}}}\right]$$

$$\sigma' = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2AhE}{WL}}\right]$$

$$\boxed{\sigma' = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2AhE}{WL}}\right]} \rightarrow \text{eq}^n \ 2.26(a), \text{ Pg } 27$$

WKT $\sigma = \frac{P}{A} = \frac{W}{A}$

$$\therefore \boxed{\sigma' = \sigma\left[1 + \sqrt{1 + \frac{2AhE}{WL}}\right]}$$

For impact deformation

x'y $\frac{L}{E}$ through out the equation.

$$\sigma' \times \frac{L}{E} = \sigma \times \frac{L}{E}\left[1 + \sqrt{1 + \frac{2AhE}{WL}}\right]$$

WKT $\delta' = \frac{\sigma' \times L}{E}$ & $\delta = \frac{\sigma \times L}{E}$

$$\therefore \delta' = \delta\left[1 + \sqrt{1 + \frac{2h}{\delta}}\right]$$

WKT

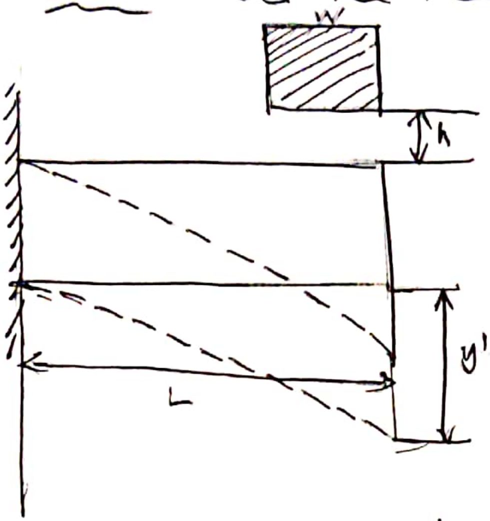
$\delta = \frac{\sigma L}{E}$

$\delta = \frac{WL}{AE}$

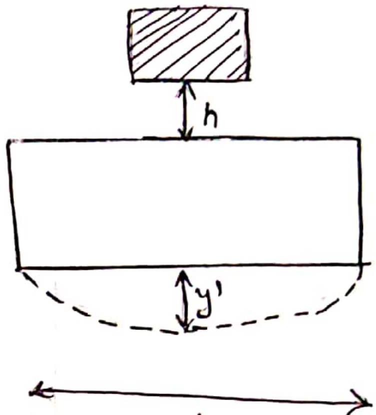
$\therefore \frac{1}{\delta} = \frac{AE}{WL}$

The quantity $\left[1+\sqrt{1+\frac{2hAE}{WL}}\right]$ i.e. $\left[1+\sqrt{1+\frac{2h}{\delta}}\right]$ is known as impact factor.

Impact stress due to bending



Cantilever support



simple support

Impact stress due to bending

$$\sigma_b' = \sigma_b\left(1+\sqrt{1+2h/y}\right) \rightsquigarrow \left[eq^n\ 2.26\,(c),\ \cancel{from}\ Pg\ 27\right]$$

Deflection under impact action due to bending.

$$y' = y\left[1+\sqrt{1+2h/y}\right] \rightsquigarrow \left[eq^n\ 2.26\,(d),\ Pg\ 28\right]$$

where σ_b' = bending stress due to impact

σ_b = bending stress due to static weight.

y' = deflection of beam due to impact.

y =

Impact stress due to torsion

Impact shear stress due to torsion

i.e., $$\tau' = \tau\left[1+\sqrt{1+\frac{2h}{\gamma\theta}}\right] \rightsquigarrow \left[eq^n\ 2.26(e),\ Pg\ 28\right]$$

- angular deformation due to impact loading

i.e. $\theta' = \theta\left[1 + \sqrt{1 + \frac{2h}{\delta\theta}}\right] \rightsquigarrow$ (eq^n 2.26(f)), Pg 28.

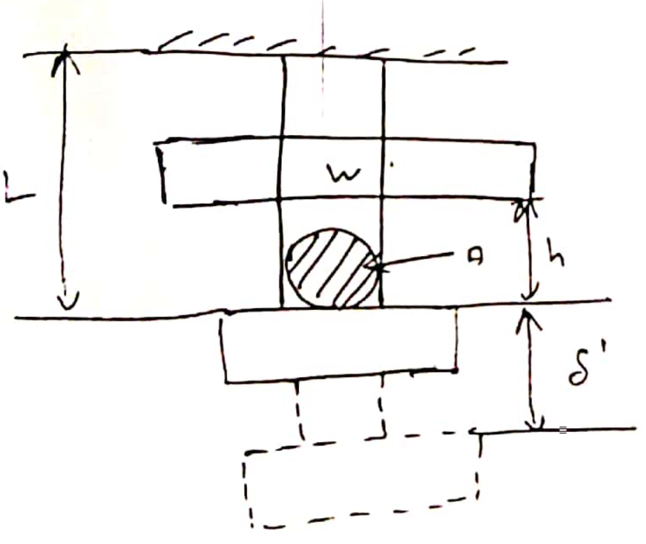
where θ = angular deflection,

h = height of force (or) weight

Problems

1) A cylindrical rod of 40mm diameter is fix at one end & as the cylindrical disc attached at the other end which receives impact by a load of 500N falling through a height of 5mm. The length of the rod is 400mm. determine i) Impact stress ii) Deformation due to impact iii) stress due to gradually apply the. iv) stress when the load is suddenly apply & v) Impact factor. Assume E=200 GPa

Sol^n



$d = 40mm$

$W = 500N$

$h = 5mm$

$L = 400mm$

Determine

i) $\sigma' = ?$

ii) $\delta' = ?$

iii) $\sigma = ?$

iv).

v) Impact factor

$E = 200\times10^9 \, N/m^2$

$= 200\times10^9\times10^{-6} \, N/mm^2$

$E = 200k \, N/mm^2$

$\boxed{E = 200\times10^3 \, N/mm^2}$

$A = \frac{\pi d^2}{4} = \frac{\pi\times(40)^2}{4}$

$\therefore A = \underline{1256.6370} \, mm^2$

i) Impact stress, $\sigma' = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hEA}{WL}}\right] \rightsquigarrow$ Eqn 2.26 (a), Pg 27

$$\sigma' = \frac{500}{1256.637}\left[1 + \sqrt{1 + \frac{2 \times 5 \times 200 \times 10^3 \times 1256.637}{500 \times 400}}\right]$$

$$\therefore \sigma' = \underline{\underline{45.0028 \text{ N/mm}^2}}$$

ii) Deformation due to impact

$$\delta' = \delta\left[1 + \sqrt{1 + \frac{2h}{\delta}}\right] \rightsquigarrow [\text{Eqn } 2.26 (b), \text{ Pg } 27]$$

wkT

$$\delta = \frac{\sigma L}{E} = \frac{WL}{AE} = \frac{500 \times 400}{1256.6371 \times 200 \times 10^3}$$

$$\delta = \underline{\underline{7.9577 \times 10^{-4}}} \text{ mm}$$

$$\delta' = 7.9577 \times 10^{-4}\left[1 + \sqrt{1 + \left(\frac{2 \times 5}{7.9577 \times 10^{-4}}\right)}\right]$$

$$\delta' = \underline{\underline{0.09 \text{ mm}}}$$

iii) stress due to gradually applied load

$$\sigma = \frac{W}{A} = \frac{500 \times 4}{\pi d^2} = \frac{500 \times 4}{\pi \times (40^2)}$$

$$\sigma = \underline{\underline{0.3978 \text{ N/mm}^2}}$$

iv) stress due to sudden load

$$\sigma' = 2\sigma \rightsquigarrow [\text{eq } 2.26 (g), \text{ Pg } 28]$$

$$= 2 \times 0.3978$$

$$\therefore \sigma' = \underline{\underline{0.7956 \text{ N/mm}^2}}$$

iv) Impact factor $= \left[1 + \sqrt{1 + \frac{2hAE}{WL}}\right]$

$= \left[1 + \sqrt{1 + \frac{2 \times 5 \times 1256.6371 \times 200 \times 10^3}{900 \times 400}}\right]$

$\therefore$ Impact factor $= 113.042$

2) A steel rod 1.5m long has to resist longitudinally an impact of 2.5kN falling under gravity at a velocity of 0.9925 m/s. The maximum computed stress is to be limited to 150MPa. Determine the diameter of the round rod. Take $E = 210 \times 10^3$ MPa.

soln

$L = 1.5m = 1500mm$

$W = 2.5 \times 10^3 N$

$v = 0.9925 m/sec$

$\sigma' = 150 MPa = 150 \times 10^6 \times 10^{-6} N/mm^2$

$\therefore \sigma' = 150 N/mm^2$

$d = ?$

$E = 210 \times 10^3 \times 10^6 \times 10^{-6} N/mm^2$

$\therefore E = 210 \times 10^3 N/mm^2$

WKT

$\sigma' = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hEA}{WL}}\right]$ ⟶ [eqn 2.26(a) pg 27]

WKT $h = \frac{v^2}{2g} = \frac{(0.9925)^2}{2 \times 9.81}$

$h = 0.05020 m$

$h = 50.2067 mm$

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$$150 = \frac{2.5\times10^3}{A}\left[1+\sqrt{1+\frac{2\times50.2067\times210\times10^3\times A}{2.5\times10^3\times1500}}\right].$$

$$\frac{150A}{2.5\times10^3} = 1+\sqrt{1+(5.6231A)}.$$

$$(0.06A-1) = \sqrt{1+(5.6231A)}$$

squaring on both side

$$(0.06A-1)^2 = 1+5.6231A$$

$$(0.06)^2A^2+\cancel{1}-2(0.06A)(1) = \cancel{1}+5.6231A$$

$$3.6\times10^{-3}A^2-5.7431A = 0$$

$$\therefore A = \underline{\underline{1595.3055\,mm^2}}$$

$$A = \frac{\pi d^2}{4}$$

$$d^2 = \frac{A\times4}{\pi}$$

$$d^2 = \frac{1595.3055\times4}{\pi}$$

$$\therefore d = \underline{\underline{45.0689\,mm}}$$

Imp

3) An unknown weight falls through 20mm on to a collar rigidly attained to the lower end of the bar 2m long & 500mm² square section. If the maximum instantaneous extention is 2mm. what is the corresponding stress & the value of unknown weight. Take E = 200GPa.

$W = ?$

$h = 20mm$

$L = 2m = 2000mm$

$A = 500mm^2$

$\delta' = 2mm$

$\sigma' = ?$

$E = 200 GPa = 200 \times 10^3 N/mm^2$

WKT $\sigma' = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hEA}{WL}}\right]$ → [eqn 2.26 (a), Pg 27]

WKT $\delta' = \frac{\sigma' \times L}{E}$

$\sigma' = \frac{\delta' \times E}{L}$

$\sigma' = \frac{2 \times 200 \times 10^3}{2000}$

$\boxed{\sigma' = 200 N/mm^2}$

(or) Instantaneous strain

$\varepsilon' = \frac{\delta'}{L}$

$\varepsilon' = \frac{2}{2000} = 10^{-3}$

WKT $E = \frac{\sigma'}{\varepsilon'}$

Instantaneous stress

$\sigma' = E \times \varepsilon'$

$\sigma' = 200 \times 10^3 \times 10^{-3}$

$\sigma' = 200 N/mm^2$

$200 = \frac{W}{500}\left[1 + \sqrt{1 + \frac{2 \times 20 \times 200 \times 10^3 \times 500}{W \times 2000}}\right]$

$200 = \frac{W}{500}\left[1 + \sqrt{1 + \frac{2 \times 10^6}{W}}\right.$

$$200 = \frac{w}{500} + \frac{w}{500}\sqrt{1 + \frac{2\times10^6}{w}}$$

$$200 - \frac{w}{500} = \frac{w}{500}\sqrt{1 + \frac{2\times10^6}{w}}$$

squaring on both side.

$$\left(200 - \frac{w}{500}\right)^2 = \frac{w^2}{(500)^2}\left(1 -\right.$$

$$\frac{1\times10^5}{w} = 1 + \sqrt{1 + \frac{2\times10^6}{w}}$$

$$\left[\frac{1\times10^5}{w} - 1\right] = \sqrt{1 + \frac{2\times10^6}{w}}$$

$$\left[\frac{1\times10^5}{w} - 1\right]^2 = 1 + \frac{2\times10^6}{w}$$

$$\frac{1\times10^{10}}{w^2} + \cancel{1} - \frac{2\times10^5}{w} = \cancel{1} + \frac{2\times10^6}{w}$$

$$\frac{1\times10^{10}}{w^2} = \frac{2\times10^6}{w} + \frac{2\times10^5}{w}$$

$$\frac{1\times10^{10}}{w^{\cancel{2}}} = \frac{2.2\times10^6}{\cancel{w}}$$

$$\frac{1\times10^{10}}{w} = 2.2\times10^6$$

$$w = \frac{1\times10^{10}}{2.2\times10^6}$$

$$\therefore W = 4545.4545\,N$$

4) A hammer of 4KN strikes the mid-span of simply supported beam of span 4m, the beam as a depth of 200mm & a width of 100mm. deter-mine the height through which the hammer can

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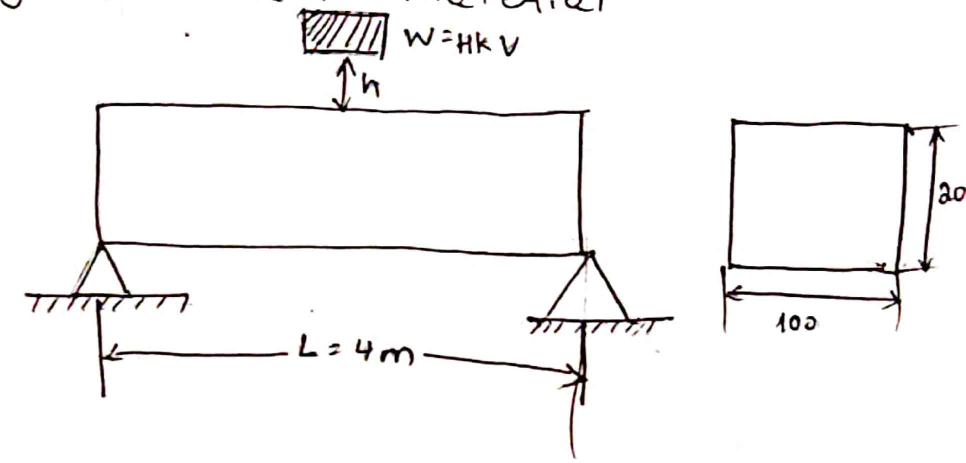
be allowed to fall, if the maximum stress in beam is limited to 100MPa, the modulus of elasticity of beam material is 206GPa.

Soln :-



$\sigma_b' = 100\ MPa = 100\ N/mm^2$

$E = 206 \times 10^3 N/mm^2$

$W = 4 \times 10^3 N$

$L = 4000mm$

$b = 100mm$

$h = 200mm$

WkT

$$\sigma_b' = \sigma_b \left(1 + \sqrt{1 + 2h/y}\right) \rightsquigarrow [\text{eqn } 2.26(d), \text{ Pg } 28]$$

Bending stress

$$\sigma_b = \frac{M \times y}{I}$$

where

$M = ?$

For simply supported beam, maximum bending moment

From pg 15 (Table 1.4)

$$M_{max} = \frac{WL}{4} = \frac{4 \times 10^3 \times 4000}{4}$$

$$\therefore M_{max} = 4 \times 10^6\ N\text{-}mm$$

From pg 12

$$y = \frac{200}{2}$$

$\therefore y = 100mm$

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(From pg 12)

moment of Inertia, $I = \frac{bh^3}{12}$

$= \frac{100 \times (200^3)}{12}$

$I = 66.667 \times 10^6 \, mm^4$

$\sigma_b = \frac{M \times y}{I} = \frac{4 \times 10^6 \times 100}{66.667 \times 10^6}$

$\sigma_b = 6 \, N/mm^2$

(From pg 15) For SSB, max deflection, $y_{max} = \frac{WL^3}{48EI}$

~~y_{max}~~

$y_{max} = \frac{4 \times 10^3 \times (4000^3)}{48 \times 206 \times 10^3 \times 66.667 \times 10^6}$

$\boxed{y_{max} = 0.3883}$

$\cancel{\sigma_b = 6 \left[1 + \sqrt{1 + \left(\frac{2 \times 200}{0.3883} \right)} \right]}$

$\therefore \cancel{\boxed{\sigma_b = 198.6674 \, N/mm^2}}$

$100 = 6 \left(1 + \sqrt{1 + \frac{2h}{0.3883}} \right)$

$\frac{100}{6} = 1 + \sqrt{1 + 5.1506h}$

$\frac{100}{6} - 1 = \sqrt{1 + 5.1506h}$

$(15.667)^2 = 1 + 5.1506h$

$\therefore h = 47.4594 \, mm$

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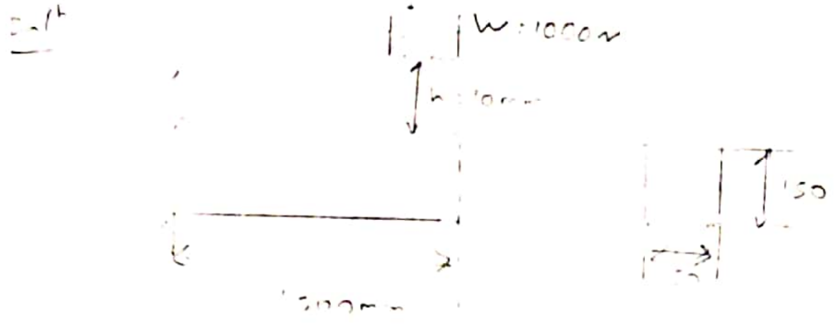
5) A cantilever beam of width 50 mm, depth 150 mm 1.5 m long it is struck by a weight of 1000. that falls from a height of 10mm at its free end. Determine the following i) Impact factor. ii) Instantaneous maximum deflection. take $E = 20.6 \times 10$ MPa
iii) Instantaneous maximum stress.
iv) Instantaneous maximum load.



$h = 10mm$

$W = 1000N$

$L = 1500mm$

$E = 20.6 \times 10^4 \, N/mm^2$

i) Impact factor = ?

ii) $y' = ?$ $y \times Im.f$

iii) $\sigma_b' = ?$ $\sigma_b \times$ Impact factor

iv) ~~Inst~~ Instantaneous load = $W \times I.F$

i) Impact factor = $\left[1 + \sqrt{1 + \frac{2h}{y}} \right]$
(pg 15)

For cantilever beam carry load at end point
(Table 1) max. deflection

From pg 12

$$I = \frac{bh^3}{12} = \frac{50 \times 150^3}{12}$$

$$I = 14.0625 \times 10^6 \, mm^4$$

$$y_{max} = \frac{WL^3}{3EI}$$

$$= \frac{1000 \times (1500)^3}{3 \times 20.6 \times 10^4 \times 14.0625 \times 10^6}$$

~~$y_{max} = 0.16504 \, mm$~~

$$y_{max} = 0.38834 \, mm$$

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$$I.F = \left[1 + \sqrt{1 + \frac{2 \times 10}{0.3883}}\right]$$

$$\boxed{I.F = 8.24614}$$

ii) $y' = y_{max} \times I.F$

$= 0.38834 \times 8.2464$

$$\boxed{y' = 3.2023 \text{ mm}}$$

iii) $\sigma_b' = \sigma_b \times I.F$

$$\sigma_b = \frac{M \times y}{I}$$

where

$M = ?$ Pg 15

For ~~simply~~ cantilever support beam, end load

$M_{max} = WL$

$= 1000 \times 1500$

$= 1.5 \times 10^6$ N-mm

$y = \frac{150}{2} = 75$ mm

~~Deflection ... max deflecth~~

~~y...=~~

$$\sigma_b = \frac{1.5 \times 10^6 \times 75}{14.0625 \times 10^6} = 8 \text{ N/mm}^2$$

$\sigma_b' = 8 \times 8.24614$

$\sigma_b' = 65.96912$ N/mm²

iv) Instantaneous load = $w \times IF$

$= 1000 \times 8.24614$

$= 8246.14 \, N$

6) A weight of 2kN falls through a height of 2mm & strikes the collar the diameter of the steel bar is 30mm & length of the bar is 50mm take E = 200 G. determine stress induced in the bar considering inertia of the bar. specific weight of the bar material is $78 kN/m^3$.

Sol^n

$W = 2kN$

$h = 2mm$

$d = 30mm$

$L = 50mm$

$E = 200 \times 10^3 \, N/mm^2$

$W' = 78 kN/m^3$

$W' = 78 \times 10^3 \times (10^{-3})^3 \, N/mm^3$

$= 78 \times 10^3 \times 10^{-9} \, N/mm^3$

$W' = 78 \times 10^{-6} \, N/mm^3$

$$\sigma' = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hEA}{WL}}\right]$$

considering Inertia

$$\sigma' = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2h_e EA}{WL}}\right]$$

note :- Effect of Inertia

when a body having weight w strikes another body which has a weight w'. according to law of collision of two perfectly inelastic bodies the impact energy wh is reduced to in nwh, when the value of n may be found using the relation

$$n = \frac{(1 + am)}{(1 + bm)^2}$$ [eq^n 2.26(i), Pg ~~8~~ 28]

where a & b are constants whose values are obtained from table 2.8 (Pg 34) from design data hand book

$$m = \frac{w'}{w}$$

WKT $n = \frac{(1 + am)}{(1 + bm)^2}$

Values of a & b from table 2.8 Pg 34

$a = 1/3$ & $b = 1/2$

WKT $m = \frac{W'}{W}$

$W' =$ specific weight $\times$ volume of bar | $= N/mm^3 \times mm^3$

$= 78 \times 10^{-6} \times$ Area $\times$ L

$= 78 \times 10^{-6} \times \frac{\pi d(30^2)}{4} \times 50$

$W' = 2.7567\ N$

$m = \frac{W'}{W} = \frac{2.7567}{2 \times 10^3} = \underline{0.001378}$

$$n = \frac{\left(1 + \frac{0.001378}{3}\right)}{\left(1 + \left(\frac{0.001378}{2}\right)\right)^2}$$

$n = \underline{0.9990}$

$h_e = n \times h$

$= 0.999 \times 2$

$\boxed{h_e = 1.998}$

$$\sigma' = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2 h_e E A}{W L}}\right]$$

$$= \frac{2000}{706.858}\left[1 + \sqrt{1 + \frac{2 \times 1.998 \times 200 \times 10^3 \times 706.858}{2000 \times 50}}\right]$$

$\boxed{\sigma' = 215.511013\ N/mm^2}$

$A = \frac{\pi d^2}{4} = \frac{\pi \times (30^2)}{4}$

$A = 706.858\ mm^2$

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Resilience & Toughness

The two Important mechanical property of the material that resist impact loading are.

i) Resilience.

ii) Toughness.

i) Resilience :- The ability of the material to observe the energy within elastic range is known as Resiliance. Since it absorbs the energy in elastic range it gives up energy when forces are removed.

Resiliance is also known as strain energy since it is measure by amount of deformation (or) strain.

$$\text{Resiliance} = \frac{(\sigma)^2 V}{2E} \rightsquigarrow [\text{eq}^n\ 2.27(b),\ \text{Pg}\ 2a]$$

where V = volume of body = $A \times l$

The strain energy stored per unit ~~area~~ volume is called modulus of resiliance.

$$\text{modulus of resiliance} = \frac{\sigma^2}{2E}$$

ii) Toughness :- The ability of material to absorb energy & deform plastically before fracture is known as Toughness.

note :- 1) Ductile materials offers greater resistance to impact than brittle material.

2) For a moving body impact energy is equal to kinetic energy

7) For a falling body impact energy is equal to potential energy

Problems

1) A steel rope runs at a speed of 2.5m/sec b/w the range of a narrow gauge roads the weight of a loaded car which must be connected to & pulled by this rope is 8kN. area of crossection of rope is 100mm² the length of rope b/w driving pulley & the point where the car is put is 300m. determine stress induced in the ~~rope~~ rope by the impact of hooking in the car. modulus of elasticity of rope is 200GPa

$v = 2.5 m/sec = 2.5\times10^3 mm/sec$

$\frac{N \frac{m^2}{s^2}}{m/s^2}$

$N-m$

$W = 8kN$

$A = 100mm^2$

$L = 300\times10^3 mm$

$\sigma' = ?$

$E = 200\times10^3 N/mm^2$

$g = 9.81\times10^3 mm/sec^2$

kinetic energy, $E_k = wh$ ~> [eqn 2.25(b) pg 27]

$= \frac{wv^2}{2g}$ ~> [eqn 2.25(a), pg 27]

$$E_k = \frac{(8\times10^3)(2.5\times10^3)^2}{2\times9.81\times10^3}$$

$E_k = 2548.4199\times10^3 N-mm$ ~> 1

WKT

Resilience $= \frac{\sigma'^2 V}{2E}$ ~> [eqn 2.27(b) pg 29]

$$= \frac{\sigma'^2 \times \overbrace{100\times300\times10^3}^{A\times L}}{2\times200\times10^3}$$ ~> 2

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equating eqn ① & ②.

$$2548.41 \times 10^3 = \frac{(\sigma')^2 \times 300 \times 10^3 \times 100}{2 \times 200 \times 10^3}$$

$$(\sigma')^2 = \frac{2 \times 200 \times 10^3 \times 2548.41 \times 10^3}{300 \times 10^3 \times 100}$$

$$\therefore \boxed{\sigma' = 184.33 \, N/mm^2}$$

2) A mass of 500 kg is being loaded by means of steel wire rope having crossection area 250mm². The velocity of the weight is 0.5m/sec when the length of extended rope is 20m, the shear gets stuck up. determine the stress induced in the rope due to sudden stoppage of the shear. take E = 190 GPa neglect the friction.

Soln

$m = 500 kg$ $\quad$ $w = 500 \times 9.81 = 4.905 \times 10^3 N$

$A = 250 mm^2$

$v = 0.5 m/sec = 0.5 \times 10^3 mm/sec$

$L = 20 m = 20 \times 10^3 mm$

$\sigma' = ?$

$E = 190 \times 10^3 N/mm^2$

$g = 9.81 m/s^2 = 9.81 \times 10^3 mm/s^2$.

E_k = kinetic energy $= Wh$ [Eqn 2.25(b), pg 27]

$$= \frac{Wv^2}{2g} \text{ [eqn 2.25(a), pg 27]}$$

$$= \frac{4.905 \times 10^3 \times (0.5 \times 10^3)^2}{2 \times 9.81 \times 10^3}$$

$E_r = 62.5 \times 10^3$ N-mm $\rightarrow$ (1)

WKT

$$\text{Resilience} = \frac{\sigma'^2 V}{2E} \quad [\text{eq}^n\ 2.27(b)\ \text{pg } 29]$$

$$= \frac{\sigma'^2 \times 250 \times 20 \times 10^3}{2 \times 190 \times 10^3} \rightarrow (2)$$

equating eq^n (1) & (2)

$$62.5 \times 10^3 = \frac{(\sigma')^2 \times 250 \times 20 \times 10^3}{2 \times 190 \times 10^3}$$

$$(\sigma')^2 = \frac{2 \times 190 \times 10^3 \times 62.5 \times 10^3}{250 \times 20 \times 10^3}$$

$$\therefore \boxed{\sigma' = 68.920 \text{ N/mm}^2}$$

3) Determine the max torsional impact that can withstand ... without permanent deformation by a 100mm cylindrical shaft 5mm long & made of a ASE-1045 ~~wrought~~ annealed steel ($\tau_y = 179.5$ MPa) & $G = 82.7$ GPa, take FoS = 3).

Sol^n

$d = 100$ mm

$L = 5m = 5000$ mm

$\tau_y = 179.5$ MPa $= 179.5$ N/mm^2

$G = 82.7$ GPa

FOS = 3

Torsional impact = Resilience

From table 2.9, Pg 34

For Torsion

modulus of Resilience = $\frac{\tau_e^2}{4G}$

WKT

$$\text{Fos} = \frac{\tau_y}{\tau_e}$$

$$\tau_e = \frac{\tau_y}{\text{Fos}} = \frac{179.5}{3}$$

$$\boxed{\tau_e = 59.833 \text{ N/mm}^2}$$

WKT

$$\text{modulus of resilience} = \frac{\tau_e^2}{4G} = \frac{(59.833)^2}{4 \times 82.7 \times 10^3}$$

$$= 0.0108$$

$$\text{Resilience} = \text{Torsional Impact} = 0.0108 \times \text{volume}$$

$$= 0.0108 \times \text{Area} \times \text{length}$$

$$= 0.0108 \times \frac{\pi d^2}{4} \times L$$

$$= 0.0108 \times \frac{(\pi \times (100^2))}{4} \times 5000$$

$$\cancel{= 425685.8046}$$

$$= 424.115 \times 10^3 \text{ N-mm}$$

The maximum torsional impact will be equal to resilience.

Design for fatigue strength :-

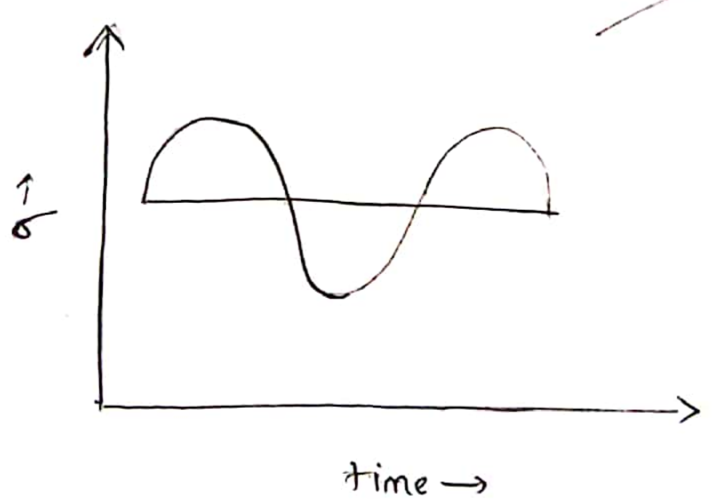
In static loading the load is apply gradually failure due to static load can be explained by simple tension test.

In fatigue loading the load is ~~re~~ repeated cyclically. The fatigue failure begins with a small ~~gr~~ ~~groove~~ crack, the crack is most likely to develop in the regions of discontinuities like Notches, holes, Grooves, region of irregularities such as machining operation like inspection mark, stamp marks, scratches etc.

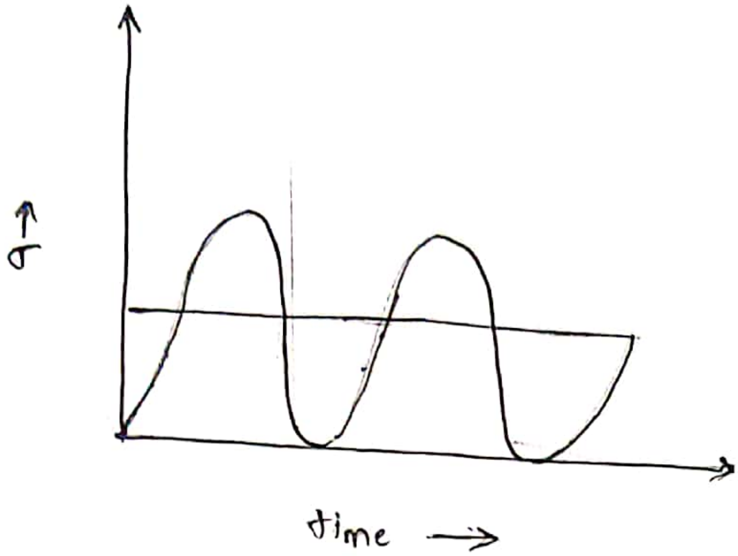
Definition

Fatigue (or) variable load :- when a load ~~...~~ (or) stress on a machine part changes in magnitude (or) direction (or) Both. the load is known as variable (or) fatigue load.

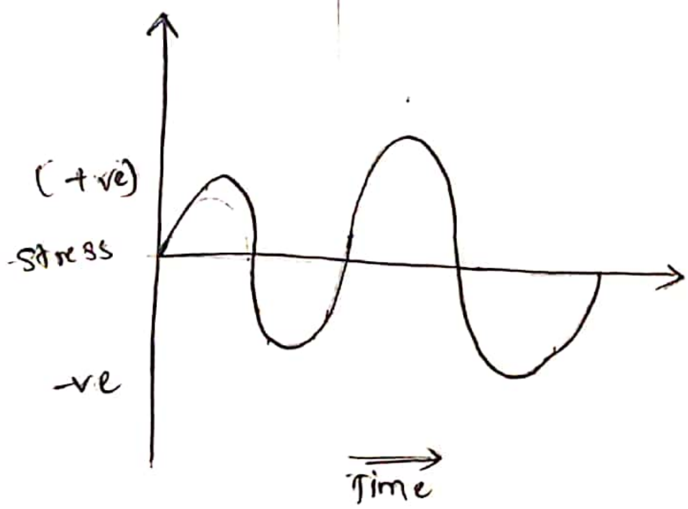
The variations are as follows.



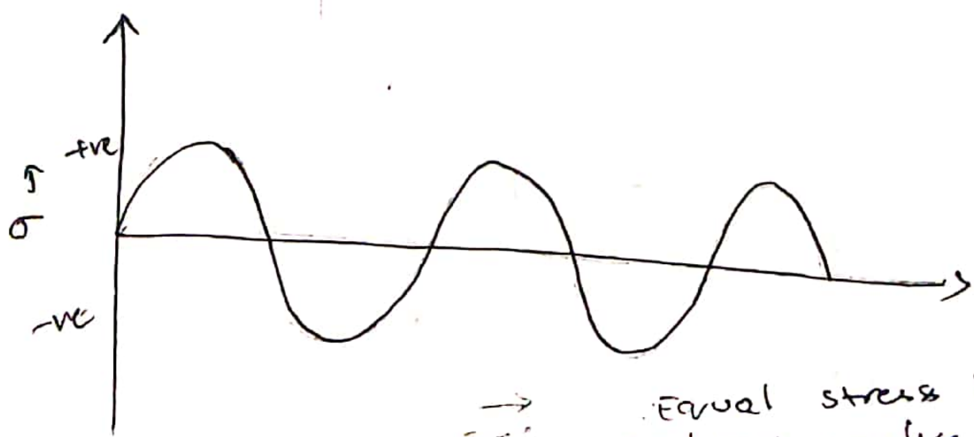
Stress variation only on Positive sides



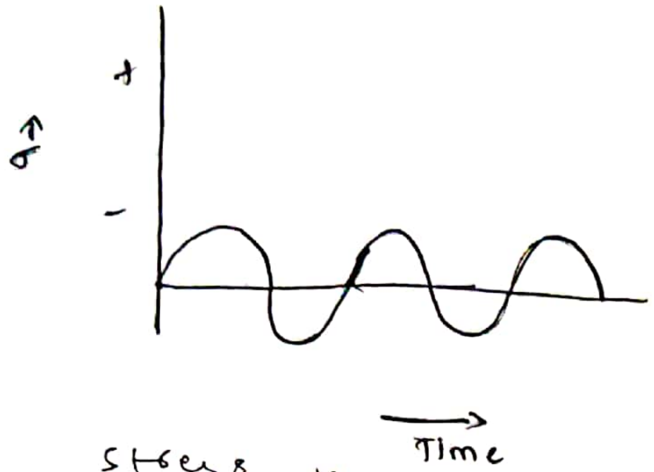
stress variation only on positive side with zero minimum stress.



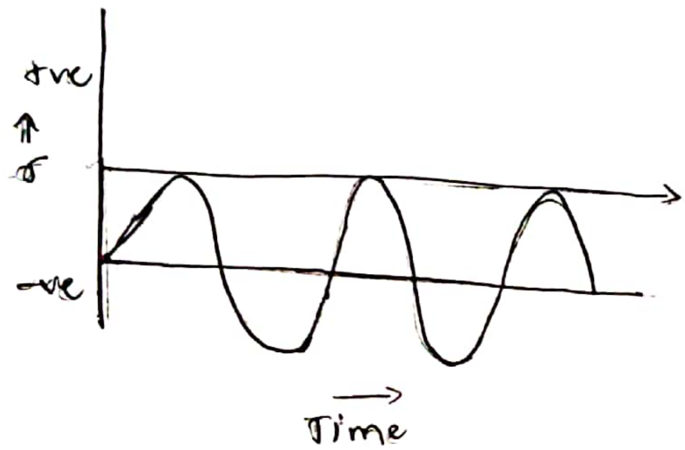
Stress variation both in negative direction



Equal stress variation both on positive & negative (completely reversed loading condition)

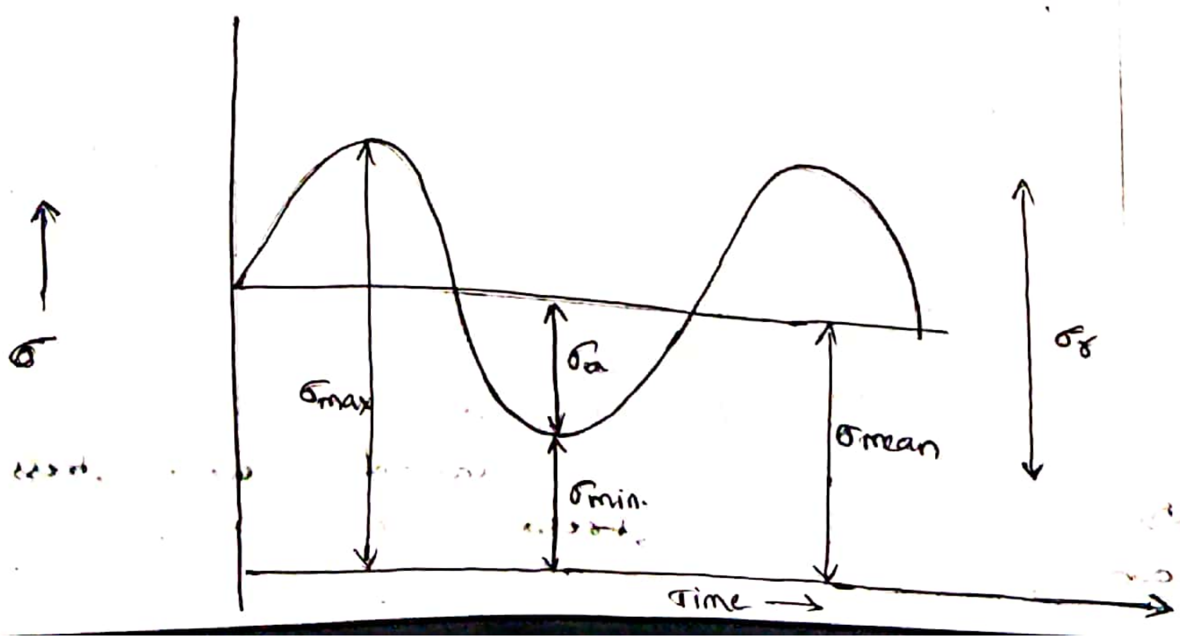


Stress variation only on negative side.



Stress variation on negative side with ~~maxtion~~ zero maximum stress.

Fluctuating stress



The stress which varies from a minimum to maximum value of same nature (tensile (or) compressive) are called fluctuating stress.

maximum stress :- The largest algebraic stress in stress cycle is known as maximum stress.

Tensile is taken as positive & compressive is taken as negative.

minimum stress :- The smallest algebraic stress in stress cycle is known as minimum stress.

mean stress :- It is the algebraic mean of maximum & minimum stress in one cycle.

$$\text{is } \sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

range of stress :- The algebraic difference b/w the maximum & minimum stress in a stress cycle is known as range of stress.

$$\text{i.e.,} = \sigma_r = \sigma_{max} - \sigma_{min}$$

stress aplitude (or) variable stress (σ_a) :- It is equal to one half of the range of stress i.e $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

stress ratio (r) :- It is defined as ratio of minimum stress to the maximum stress in the stress cycle.

$$\text{i.e.,} \quad r = \frac{\sigma_{min}}{\sigma_{max}}$$

Applitude ratio (A) :- It is defined as the ratio of variable stress to the mean stress in a stress cyle.

$$A = \sigma_a / \sigma_m$$

Repeated stress:- The stress which varies from zero to a certain maximum value are called Repeated stress.

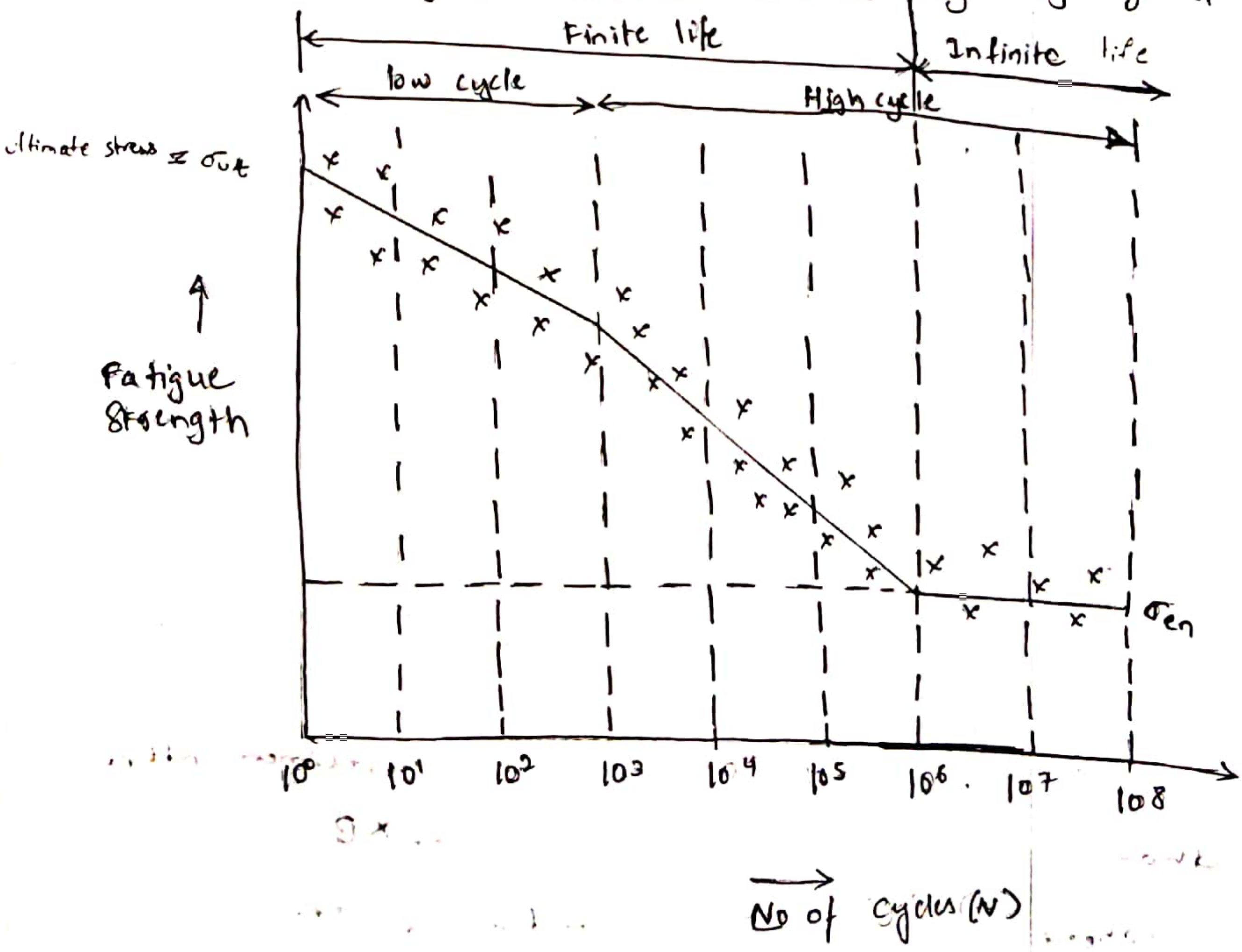
~~Aptitude (or) complet~~

Alternating (or) completely reversed stress:- The stress which vary from a minimum value to a maximum value of the opposite nature (minimum compression to maximum tension & vice versa) is called reversed (or) alternating (or) cyclic stress.

Imp (explain)

SN - Diagram & Endurance limit :-

S-N curve is the graphical representation of stress amplitude (max Applied stress) V/s the no of stress cycle (N) before the fatigue failure on a log log graph paper.



the ordinate of S-N diagram is called the fatigue strength & a statement of which is always accompanied by a statement of no of cycle (N).

- There are two regions in the curve namely low cycle fatigue & high cycle fatigue.

- Any fatigue failure when the no of stress cycles are less than 1,000 (10^3) is called low cycle fatigue. Any fatigue failure when the no of stress cycles are more than 1000 (10^3) is called high cycle fatigue.

- Fatigue strength at 10^6 cycles of reversible loading is termed as endurance limit & is designated by σ_{en}.

- Endurance (or) fatigue limit of a material is defined as the maximum aplitude of a completely reversed stress cycle the standard specimen can sustain for an unlimited number of cycles without fatigue failure.

- Fatigue life is defined as the no of stress cycle that the standard specimen can complete during a test before the apperance of first fatigue crack.

Effects of factors on endurance limit

1) surface condition :- Endurance limit for a specimen considering the effect

$$\text{Endurance limit for a specimen} = \sigma_{en} \times C$$

where C is surface correction co-efficient.

2) size effect :- Endurance limit for a specimen other than standard diameter $= \sigma_{en} \times B$

where B is size correction co-efficient.

3) effect of load on endurance limit :- Endurance limit for specimen other than reverse bending load = $\sigma_{en} \times A$

when A is load correction co-efficient.

4) effect of reliability on endurance limit :- The co-efficient of reliability (e_r) depends on the reliability that is consider in the design of the component. The reliability of the fatigue is 50%.

If the reliability is other than 50% then the

endurance limit = $\sigma_{en} \times e_r$

5) effect of stress concentration on endurance limit :- All discontinuities effect the endurance limit of a machine part. The endurance limit is reduce due to stress concentration.

The stress concentration factor used for cyclic loading is less than the theoretical stress concentration due to notch sensitivity of the material

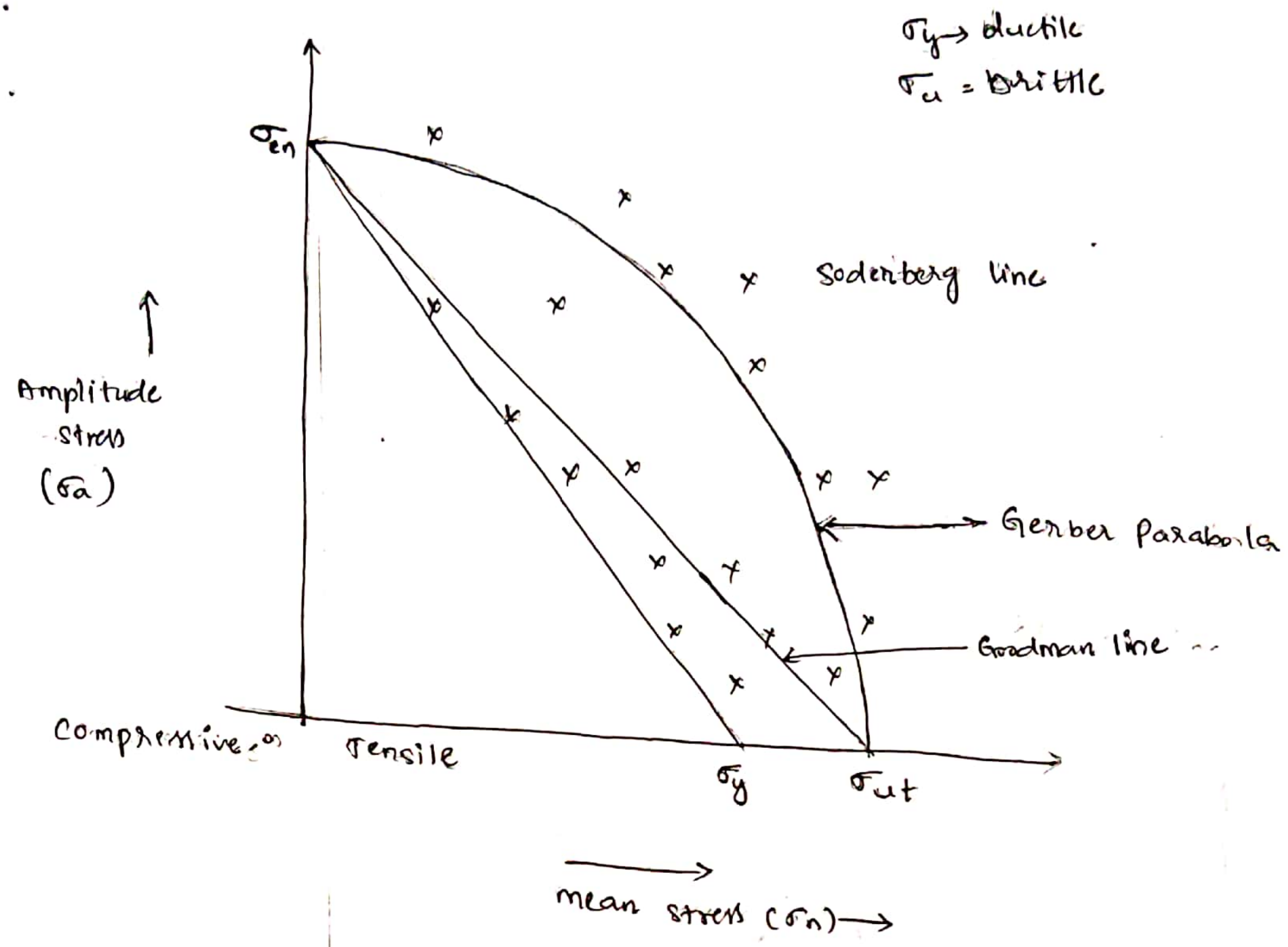
Endurance limit of a specimen will be equal to $\frac{\sigma_{en}}{K_{tf}}$

$$= \frac{\sigma_{en}}{K_{tf}} = K_d \, \sigma_{en}$$

considering various modifying factor endurance limit will be equal to $= \sigma_{en} \times ABC \times K_d$

where $K_d = \frac{1}{K_{tf}}$

fatigue strength under fluctuating loads



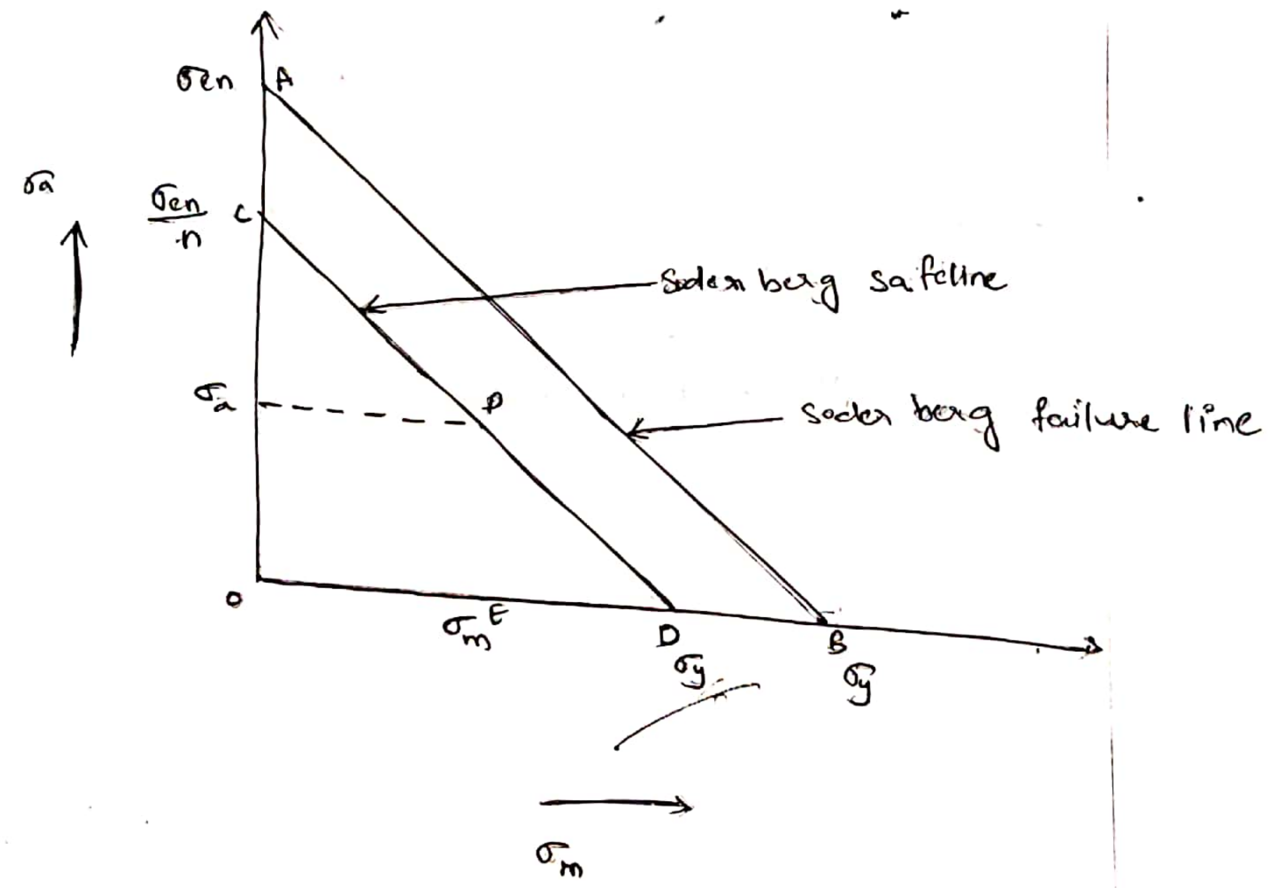
- when a component is subjected fatigues (fluctuating stress)
- The stress are resolved into two part
  i) mean stress
  ii) Amplitude stress.
- In the diagram means stress are ploted on the horizontal axis with tensile stress on right & compression on left side of the origin. Stress Amplitude is plotted on the vertical axis.
- when $\sigma_a = 0$ the load is Purely static & hence the criterian for failure is σ_u (or) σ_y.
- when $\sigma_m = 0$, the stress is completely reversible & hence the criterian of failure is endurance limit (σ_{en}).

The parabolic failure curve was proposed by Gerber, a straight line joining σ_{en} to σ_u is called goodman line, while the line joining σ_{en} to σ_y is called soderberg line.

Imp Derivation

Soderberg criterion

According to soderberg criterian the straight line joining the endurance limit & the yield strength is taken as failure line as shown in figure



In the figure the line AB termed as soderberg failure lines.

Considering suitable FOS (n) the line CD can be drawn parallel to line AB. this line is called soderberg safe line.

Consider a point 'P' on a soderberg safe line as sh...

at this σ_m & σ_a be the mean & amplitude stress at Point. ~~draw~~ Draw PE $\perp^{r}$ to OB.

considering similar Δ^{le}

COD & PED

$$\frac{PE}{CO} = \frac{ED}{OD}$$

$$\frac{PE}{CO} = \frac{OD - OE}{OD}$$

$$\frac{PE}{CO} = \cancel{\frac{OD}{OD}} - \frac{OE}{OD}$$

$$\frac{PE}{CO} = 1 - \frac{OE}{OD}$$

$$\frac{\sigma_a}{\frac{\sigma_{en}}{n}} = 1 - \frac{\sigma_m}{\sigma_y/n}$$

$$n.\frac{\sigma_a}{\sigma_{en}} + n.\frac{\sigma_m}{\sigma_y} = 1$$

$$\therefore \boxed{\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}}$$

considering the effect of stress concentration & the three Important modifying factor.

$$\boxed{\frac{K_{tf} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}}$$

2nd method

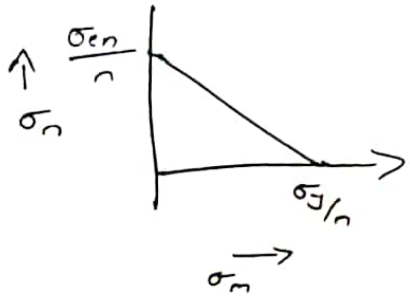
Apply the straight line equation of the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

a & b are intercepts of the line in x & y direction respectively.

Applying the above relation. the eqn of soderberg line is given by.

$$\frac{\sigma_m}{\sigma_y/n} + \frac{\sigma_a}{\sigma_{en}/n} = 1$$



$$\boxed{\frac{\sigma_m}{\sigma_y} + \frac{\sigma_a}{\sigma_{en}} = \frac{1}{n}}$$

Notes

1) Relationship b/w tensile strength & endurance limit.

a) For steel :- $\sigma_{en} = 0.5\,\sigma_{ut}$

b) For castiron :- $\sigma_{en} = 0.4\,\sigma_{ut}$ → eqn 2.13 (a, b & c) Pg

c) For Aluminium :- $\sigma_{en} = 0.3\,\sigma_{ut}$

' notch sensitivity index (q) :-

$$q = \frac{k_{tf} - 1}{k_t - 1} \rightsquigarrow [\text{eq } 2.12(a)]$$

i.e., $k_{tf} = 1 + q(k_t - 1)$

> soderberg eqn for ductile material

$$\frac{k_{tf}\,\sigma_a}{ABC\,\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \rightsquigarrow (\text{eq}^n\ 2.21\ (c)\ \text{pg } 25)$$

Soderberg eqn for ductile material in shear.

$$\frac{K_{sf}\,\tau_a}{ABC\,\sigma_{en}} + \frac{\tau_m}{\tau_y} = \frac{1}{n} \quad \sim [\text{eq}^n\ 2.21(c)\ \text{pg}\ 25]$$

5) Goodman's eqn for brittle material

$$\frac{K_{tf}\,\sigma_a}{ABC\,\sigma_{en}} + K_t\,\frac{\sigma_m}{\sigma_u} = \frac{1}{n} \quad \sim [\text{eq}^n\ 2.21(b)\ \text{pg}\ 25]$$

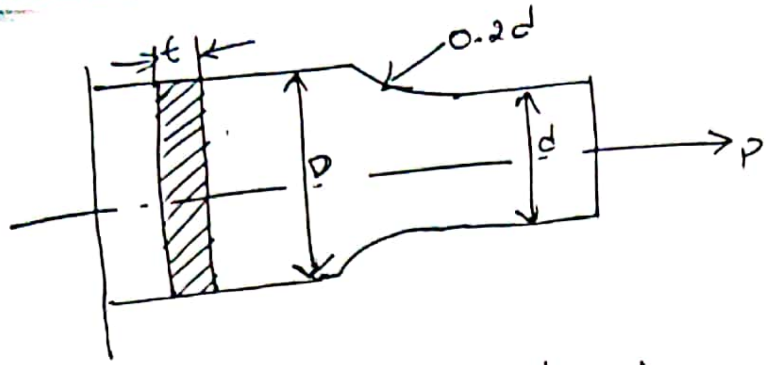
For ductile material the yield stress the yield stress is usually taken as the failure stress hence Soderberg eqn (or) line is normally used.

For brittle material the yield point is not well-defined, the ultimate stress is taken as failure stress therefore Goodman line is used.

Problem

1) A component as shown in figure is made of C-40 steel that is to be design for dynamic strength based on FOS 2. Determine the thickness in the following cases.
i) P varies from 10 kN to 20 kN
ii) P varies from -10 kN to 20 kN
iii) P varies from -5 kN to 20 kN
iv) P repeated with max. load 20 kN.

given, d = 50 mm & D/d = 2

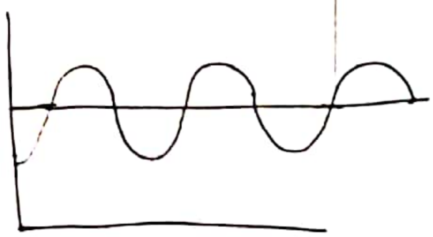


soln :- since the given material is ductile & it is subjected to dynamic loading. soderberg design equation is used.

wkT

$$\frac{K_{tf} \times \sigma_a}{ABC\,\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \rightarrow [\text{eqn } 2.26(c), \text{ Pg } 25]$$

i) P varies from 10kN to 20kN



$P_{max} = 20kN$

$P_{min} = 10kN$

$$\sigma_{max} = \frac{P_{max}}{A}$$

for bar

when $A = t \times d$ ⇝ [from Pg 38]

$$\sigma_{max} = \frac{20 \times 10^3}{t \times d} = \frac{20 \times \cancel{10^3}}{t \times 50}$$

$$\therefore \boxed{\sigma_{max} = \frac{400}{t}}$$

$$\sigma_{min} = \frac{P_{min}}{A} = \frac{10 \times 10^3}{t \times d} = \frac{10 \times 10^3}{t \times 50}$$

$$\therefore \boxed{\sigma_{min} = \frac{200}{t}}$$

$$\text{mean stress } (\sigma_m) = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$= \frac{\left(\frac{400}{t}\right) + \left(\frac{200}{t}\right)}{2}$$

$$\therefore \boxed{\sigma_m = \frac{300}{t}}$$

Amplitude stress $(\sigma_a) = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

$$= \dfrac{\left(\frac{400}{t}\right) - \left(\frac{300}{t}\right)}{2}$$

$$\therefore \boxed{\sigma_a = \frac{100}{t}}$$

For C-40 stress From table 1.18 pg 464

Yield stress, $\sigma_y = 324\ N/mm^2$

Ultimate tensile strength $= \sigma_{ut} = 570\ N/mm^2$

WKT endurance strength, $\sigma_{en} = 0.5 \times \sigma_{ut}$

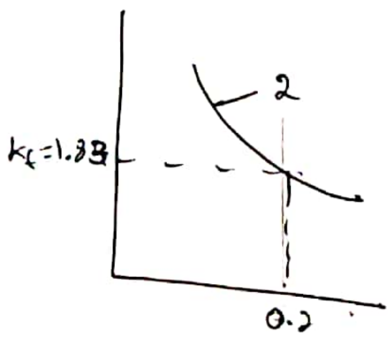
$= 0.5 \times 570$

$$\therefore \boxed{\sigma_{en} = 285\ N/mm^2}$$

WKT $K_{tf} = 1 + q(K_t - 1)$

To find K_t

From the fig 2.16 pg 38



$\frac{r_i}{b} = \frac{r}{d} = \frac{0.2\cancel{d}}{\cancel{d}} = 0.2$

$\frac{B}{b} = \frac{D}{d} = 2$

$K_t = 1.83$

To find 'q'

from the fig 2.31 Pg 46

$0.2d = 0.2 \times 50 = 10$

$\boxed{q = 0.95}$ (steel & annealed)

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then $K_{ff} = 1 + q(K_t - 1)$

$= 1 + 0.95(1.83 - 1)$

$K_{ff} = \underline{1.7885}$

From Pg 25

Load correction factor, $A = 0.7$ (Axial load)

size correction factor, $B = 0.8$ (Assume)

surface correction factor, $C = 0.85$ (Assume)

substitute the all the values in eqn

$$\frac{K_{ff} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1.78 \times (100/t)}{0.7 \times 0.8 \times 0.85 \times 285} + \frac{300/t}{324} = \frac{1}{2}$$

$$\frac{1.3183}{t} + \frac{0.9259}{t} = \frac{1}{2}$$

$$\cancel{\frac{2.2380}{t} = \frac{1}{2}}$$

$$\cancel{t = \frac{2.2380}{0.5}}$$

$$\cancel{\therefore t = 4.478\ mm}$$

$$\frac{2.2442}{t} = \frac{1}{2}$$

$\therefore t = \underline{4.4885\ mm}$

b) $\sigma_{max} = \frac{P_{max}}{A} = \frac{20 \times 10^3}{t \times d} = \frac{20 \times 10^3}{t \times 50} = \frac{400}{t}$

$\sigma_{min} = \frac{P_{min}}{Area} = \frac{-10 \times 10^3}{t \times d} = \frac{-10 \times 10^3}{t \times 50} = \frac{-200}{t}$

mean stress (σ_m) = $\dfrac{\sigma_{max} + \sigma_{min}}{2}$

$$= \frac{\left(\frac{400}{t}\right) + \left(\frac{-200}{t}\right)}{2}$$

$$\boxed{\sigma_m = \frac{100}{t}}$$

Amplitude stress (σ_a) = $\dfrac{\sigma_{max} - \sigma_{min}}{2}$

$$= \frac{\left(\frac{400}{t}\right) - \left(\frac{-200}{t}\right)}{2}$$

$$\boxed{\sigma_a = \frac{300}{t}}$$

$\sigma_y = 324\,MPa$ $\quad\quad$ $\sigma_{en} = 285\,N/mm^2$

$K_{tf} = 1.7885$ $\quad\quad$ $n = 2$

$$\frac{K_{tf} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1.7885 \times \left(\frac{300}{t}\right)}{0.7 \times 0.8 \times 0.85 \times 285} + \frac{\frac{100}{t}}{324} = \frac{1}{2}$$

$$\frac{3.955108}{t} + \frac{0.308641}{t} = \frac{1}{2}$$

$$\therefore \boxed{t = 8.527498\,mm}$$

c) P varies from $-5\,kN$ to $20\,kN$

$P_{max} = 20\,kN$

$P_{min} = -5\,kN$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{20 \times 10^3}{t \times d} = \frac{20 \times 10^3}{t \times 50} = \frac{400}{t}$$

$$\sigma_{min} = \frac{P_{min}}{A} = \frac{-5 \times 10^3}{t \times d} = \frac{-5 \times 10^3}{t \times 50} = \frac{-100}{t}$$

mean stress $(\sigma_m) = \dfrac{\sigma_{max} + \sigma_{min}}{2}$

$= \dfrac{\left(\frac{400}{t}\right) + \left(\frac{-100}{t}\right)}{2}$

$\therefore \boxed{\sigma_m = \frac{150}{t}}$

Amplitude stress $(\sigma_a) = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

$= \dfrac{\left(\frac{400}{t}\right) - \left(\frac{-100}{t}\right)}{2}$

$\therefore \boxed{\sigma_a = \frac{250}{t}}$

$\sigma_y = 324\,MPa \qquad \sigma_{en} = 285\,N/mm^2 \qquad n = 2$

$K_{tf} = 1.7885$

$$\frac{K_{tf} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1.7885 \times \left(\frac{250}{t}\right)}{0.7 \times 0.8 \times 0.85 \times 285} + \frac{\frac{150}{t}}{324} = \frac{1}{2}$$

$$\frac{3.29592}{t} + \frac{0.46296}{t} = \frac{1}{2}$$

$\therefore \boxed{t = 7.5177\,mm}$

) P is repeated with the max load 20kN

$P_{max} = 20\,kN$

$P_{min} = 0\,kN$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{20\times10^3}{t\times d} = \frac{20\times10^3}{t\times50} = \frac{400}{t}$$

$$\sigma_{min} = 0$$

$$\text{mean stress } (\sigma_m) = \frac{\sigma_{max}+\sigma_{min}}{2}$$

$$= \frac{\left(\frac{400}{t}\right)+0}{2}$$

$$= \underline{\underline{\frac{200}{t}}}$$

$$\text{Amplitude stress } (\sigma_a) = \frac{\sigma_{max}-\sigma_{min}}{2}$$

$$= \frac{\left(\frac{400}{t}\right)-0}{2}$$

$$= \underline{\underline{\frac{200}{t}}}$$

$\sigma_y = 324\,\text{MPa}$ $\quad\quad$ $\sigma_{en} = 285\ \text{N/mm}^2$

$K_{ff} = 1.7885$ $\quad\quad$ $n=2$

$$\frac{K_{f}\times\sigma_a}{ABC\,\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1.7885\times\left(\frac{200}{t}\right)}{0.7\times0.8\times0.85\times285} + \frac{(200/t)}{324} = \frac{1}{2}$$

$$\frac{2.63673}{t} + \frac{0.61728}{t} = \frac{1}{2}$$

$$\boxed{t = 6.50804\,\text{mm}}$$

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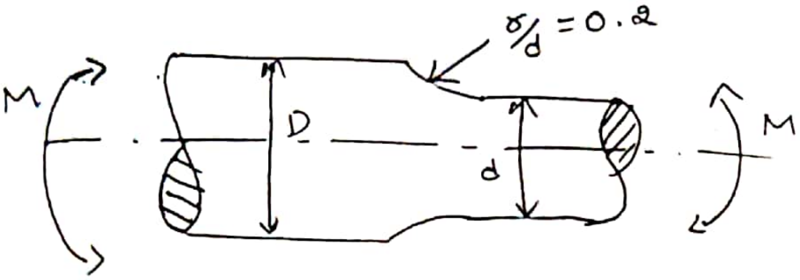
2) A step shaft as shown in figure is subjected to a Bending moment of 500N-m clockwise & ~~300~~ 300N-m counter clockwise. Determine the diameter of the shaft based on FOS = 1.8. If it is made of C-40 steel the notch sensitivity index of the material is 0.85. Given $D/d = 2$,



Soln:-

$M_{max} = 500 \, N\text{-}m \, (cw)$

$M_{min} = -300 \, N\text{-}m \, (ccw)$

$\frac{D}{d} = 2$

$n = 1.8$

material C-40 steel

From Table 1.8 Pg 464

For C-40 steel

$\sigma_{UE} = 570 \, N/mm^2$

$\sigma_y = 324 \, N/mm^2$

wk

$\sigma_{en} = 0.5 \times \sigma_u$

$= 0.5 \times 570$

$\therefore \sigma_{en} = 285 \, N/mm^2$

$q = 0.85$

Since the given material is ductile, the design eqn is soderberg equation i.e $\frac{K_{ff} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$ → [eqn 2.21(c) pg 25]

minimum stress, $\sigma_{min} = \frac{32 M_{min}}{\pi d^3}$ → (for circular)

$$= \frac{32 \times (-300) \times 10^3}{\pi \times d^3}$$

$$= \frac{-9600 \times 10^3}{\pi d^3}$$

$$\therefore \boxed{\sigma_{min} = \frac{-3.055 \times 10^6}{d^3}}$$

maximum stress, $\sigma_{max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 500 \times 10^3}{\pi d^3}$

$$\therefore \boxed{\sigma_{max} = \frac{5.09129 \times 10^6}{d^3}}$$

mean stress = $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$$= \frac{\frac{5.09129 \times 10^6}{d^3} - \frac{3.055 \times 10^6}{d^3}}{2}$$

$$\boxed{\sigma_m = \frac{1.01895 \times 10^6}{d^3}}$$

amplitude stress = $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

$$= \frac{\left(\frac{5.09129 \times 10^6}{d^3}\right) + \left(\frac{3.055 \times 10^6}{d^3}\right)}{2}$$

$$\boxed{\sigma_a = \frac{4.07395 \times 10^6}{d^3}}$$

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To find K_{tf}

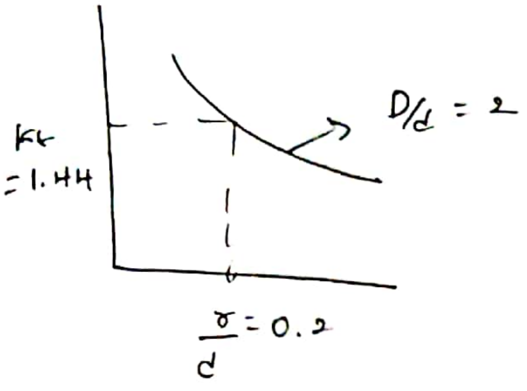
$$K_{tf} = 1 + q(K_t - 1)$$

To find K_t

from 2.25 pg 43

$$K_{tf} = 1 + 0.85\,(1.44 - 1)$$

$$\boxed{K_{tf} = 1.374}$$



A = 1.0 (reversed Bending)

B = 0.8 (Assume)

C = 0.85 (assume)

substitute all values in eqn of soderberg

$$\frac{K_{tf} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1.374 \times \left(\frac{4.07395 \times 10^6}{d^3}\right)}{1 \times 0.8 \times 0.85 \times 285} + \frac{\left(\frac{1.01895 \times 10^6}{d^3}\right)}{324} = \frac{1}{1.8}$$

$$\frac{28.8834 \times 10^3}{d^3} + \frac{3.14490 \times 10^3}{d^3} = \frac{1}{1.8}$$

$$\frac{32.0283 \times 10^3}{d^3} = \frac{1}{1.8}$$

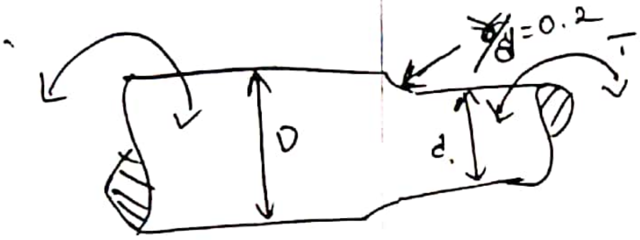
$$\boxed{d = 38.6309\,mm}$$

WKT

$$\frac{D}{d} = 2$$

$$D = 2d = 2 \times 38.6309$$

3) Determine the diameter of the stepped shaft for the given figure. If the shaft is subjected to a twisting movement of +500 N-m & -300 N-m also determine the fillet radius of FOS = 1.8 take the shaft material as C-40 steel.



$r = ?$

$d = ?$

$D/d = 2$

Sol^n :-

$T_{max} = 500\,N\text{-}m = \underline{500 \times 10^3\,N\text{-}mm}$

$T_{min} = -300\,N\text{-}m = \underline{-300 \times 10^3\,N\text{-}mm}$

$n = 1.8$

material as C-40 steel

From table 1.8 pg 464 $\sigma_y = 324\,N/mm^2$

$\sigma_u = 570\,N/mm^2$

$\sigma_{en} = 0.5 \times \sigma_u$

$= 0.5 \times 570$

$\boxed{\sigma_{en} = 285\,N/mm^2}$

IMP

$\therefore \tau_y = \frac{\sigma_y}{2} = \frac{324}{2} = \underline{162\,N/mm^2}$

soderberg equation for twisting moment

$$\frac{K_{sf} \times \tau_a}{ABC \times \sigma_{en}} + \frac{\tau_m}{\tau_y} = \frac{1}{n} \rightarrow [eq^n\ 2.21(c)\ pg\ 25]$$

max. shear stress (For circular).

$$\tau_{max} = \frac{16\,T_{max}}{\pi d^3} = \frac{16 \times 500 \times 10^3}{\pi \times d^3} = \frac{2.5464 \times 10^6}{d^3}$$

n. shear stress

$$\tau_{min} = \frac{16 \times T_{min}}{\pi d^3} = \frac{16 \times -300 \times 10^3}{\pi d^3} = -\frac{1.5278 \times 10^6}{d^3}$$

Amplitude shear stress $= \tau_a = \dfrac{\tau_{max} - \tau_{min}}{2}$

$$= \frac{\left(\dfrac{2.5464 \times 10^6}{d^3}\right) + \dfrac{1.5278 \times 10^6}{d^3}}{2}$$

$$\boxed{\sigma_a = \frac{2.0371 \times 10^6}{d^3}}$$

mean shear stress $= \tau_m = \dfrac{\tau_{max} + \tau_{min}}{2}$

$$= \frac{\left(\dfrac{2.5464 \times 10^6}{d^3}\right) - \left(\dfrac{1.5278 \times 10^6}{d^3}\right)}{2}$$

$$\boxed{\tau_m = \frac{509.3 \times 10^3}{d^3}}$$

To find K_{sf}

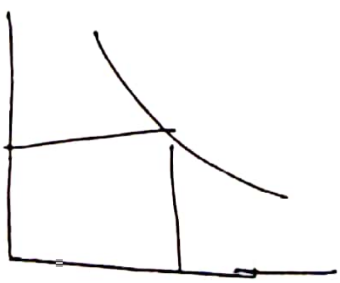
$$K_{sf} = 1 + q\,(K_t - 1)$$

$q = 0.85$ (assume)

To find K_t

From Fig 2.27 pg 44

$K_t = 1.24$



$r/d = 0.2$

$$K_{sf} = 1 + 0.85\,(1.24 - 1)$$

$$\boxed{K_{sf} = 1.204}$$

$A = 0.50$ (torsional loading)

$B = 0.8$

$C = 0.8$

WKT

$$\frac{K_{sf} \times \tau_a}{ABC \times \sigma_{en}} + \frac{\tau_m}{\tau_y} = \frac{1}{n}$$

$$\frac{1.204 \times \left(\frac{2.0371 \times 10^6}{d^3}\right)}{0.5 \times 0.8 \times 0.8 \times 285} + \frac{\left(\frac{509.3 \times 10^3}{d^3}\right)}{162} = \frac{1}{1.8}$$

$$\frac{26.8932 \times 10^3}{d^3} + \frac{3.1438 \times 10^3}{d^3} = \frac{1}{1.8}$$

$$\therefore \boxed{d = 37.8131 \text{ mm}}$$

$$\frac{r}{d} = 0.2$$

$$r = 0.2 \times d$$

$$r = 0.2 \times 37.8131$$

$$\therefore \boxed{r = 7.5626 \text{ mm}}$$

$$\frac{D}{d} = 2$$

$$D = 2 \times d$$

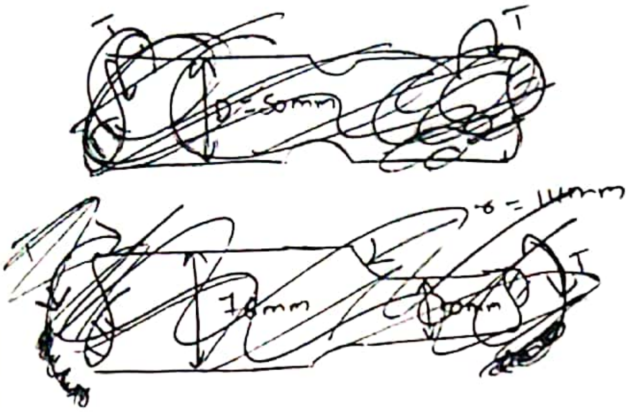
$$D = 2 \times 37.8131$$

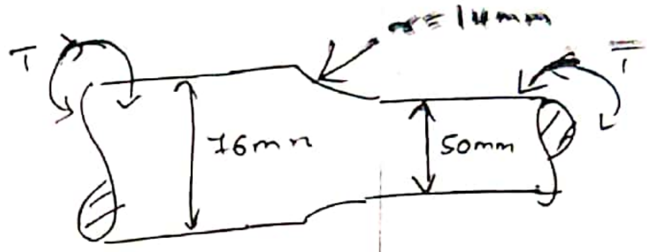
$$\therefore \boxed{D = 75.6262 \text{ mm}}$$

4) A cast iron shaft ($\sigma_u = 180$ MPa) is subjected to a torsional load which is completely reversed, the shaft has to carry this load for infinite length of time, the shaft 50mm diameter is joined to another of 16mm diameter with a fillet radius of 14mm what is the max torque the shaft can transmit

ased on FOS of 2.

,1h





Given material as cast iron

$\sigma_u = 180\ N/mm^2$

$D = 76mm$ & $\underline{n = 2}$

$d = 50mm$

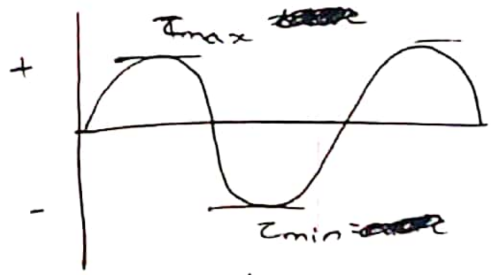
$r = 14mm$

For completely reversed

consider, $T_{max} = -T_{min}$

$T_{max}/T_{min} = ?$

completely revers



For Brittle material (CI)

the design equation ~~is~~, Goodman equation.

$$\frac{K_{sf} \times \tau_a}{ABC \times \sigma_{en}} + K_t \frac{\tau_m}{\tau_y} = \frac{1}{n} \rightarrow [2.21(b), Pg\ 25]$$

max. shear stress $= \tau_{max} = \frac{16 T_{max}}{\pi d^3}$

min. shear stress $= \tau_{min} = \frac{16 T_{min}}{\pi d^3} = -\frac{16 T_{max}}{\pi d^3}$

Mean shear stress $= \tau_m = \frac{\tau_{max} + \tau_{min}}{2}$

$$= \frac{\left(\cancel{\frac{16 T_{max}}{\pi d^3}}\right) - \left(\cancel{\frac{16 T_{max}}{\pi d^3}}\right)}{2}$$

$\boxed{\tau_m = 0}$

amplitude shear stress : $\tau_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

$$= \frac{\left(\frac{16 T_{max}}{\pi d^3}\right) + \left(\frac{16 T_{max}}{\pi d^3}\right)}{2}$$

$$= \frac{\cancel{2}}{\cancel{2}}\left(\frac{16 T_{max}}{\pi d^3}\right)$$

$$\therefore \boxed{\tau_a = \frac{16 T_{max}}{\pi d^3}}$$

To find K_{sf}

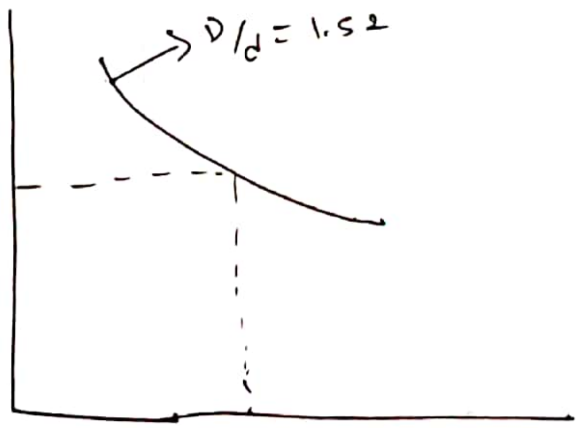
$K_{sf} = 1 + q(k_t - 1)$

To find k_t

From Fig 2.21 pg 44

$\frac{r}{d} = \frac{14}{50} = 0.28$

$\frac{D}{d} = \frac{76}{50} = 1.52$



k_t

$k_t = 1.18$

$q = 1$ (assume) & $k_t = 1.18$

$K_{sf} = 1 + (k_t - 1)$

$= 1 + (1.18 - 1)$

$\boxed{K_{sf} = 1.18}$

$\sigma_u = 180\ N/mm^2$

$\sigma_{en} = 0.4\sigma_u$

$= 0.4 \times 180$

$\boxed{\sigma_{en} = 72\ N/mm^2}$

(Bending) $A = 0.5$ $B - 0.8$ & $C = 0.8$

substitute the all values in eqn of Goodman's eqn

$$\frac{1.18 \times 16 T_{max}}{0.5 \times 0.8 \times 0.8 \times 72 \times \pi d^3} + 1.18 \left(\frac{0}{\sigma_y}\right) = \frac{1}{2}$$

$$\frac{0.2608 T_{max}}{(50^3)} + 0 = \frac{1}{2}$$

$$10^{-6} \times 2.086698 T_{max} = \frac{1}{2}$$

$$T_{max} = \frac{1}{2 \times 2.086698 \times 10^{-6}}$$

$$T_{max} = 239.61299 \times 10^3 \text{ N-mm}$$

$$\therefore \boxed{T_{max} = 239.61 \text{ N-m}}$$

~~Tmax~~

$$\therefore \boxed{T_{min} = -239.61 \text{ N-m}}$$

5) A steel rod SAE 9260 oil qunched ($\sigma_u = 1089.5$ MPa, $\sigma_y = 689.4$ MPa, $\sigma_{en} = 427.6$ MPa) is subiected to a tensile load which varies from 120kN to 40kN design the safe diameter of the rod using soderberg criteria. adopt factor of safety as 2, stress concentration factor unity & correction factors for load, size & surface 0.75, 0.85 & 0.91 respectively.

soln

$\sigma_u = 1089.5$ N/mm² $d = ?$

$\sigma_y = 689.4$ N/mm² $n = 2$

$\sigma_{en} = 427.6$ N/mm² $K_{tf} = 1$

$P_{max} = 120$ kN $A = 0.75$

$P_{min} = 40$ kN $B = 0.85$

$C = 0.91$

WKT

$$\frac{K_{fL} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad [eq^n\ 2.21(c),\ pg\ 25) \rightarrow ①$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{120 \times 10^3}{\pi d^2/4} = \frac{480 \times 10^3}{\pi d^2} = \frac{152.7887 \times 10^3}{d^2}$$

$$\sigma_{min} = \frac{P_{min}}{A} = \frac{40 \times 10^3 \times 4}{\pi d^2} = \frac{50.9295 \times 10^3}{d^2}$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{\left(\frac{152.7887 \times 10^3}{d^2}\right) + \left(\frac{50.9295 \times 10^3}{d^2}\right)}{2}$$

$$\boxed{\sigma_m = \frac{101.8591 \times 10^3}{d^2}}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\left(\frac{152.7887 \times 10^3}{d^2}\right) - \left(\frac{50.9295 \times 10^3}{d^2}\right)}{2}$$

$$\boxed{\sigma_a = \frac{50.9296 \times 10^3}{d^2}}$$

substitute in eq^n ①

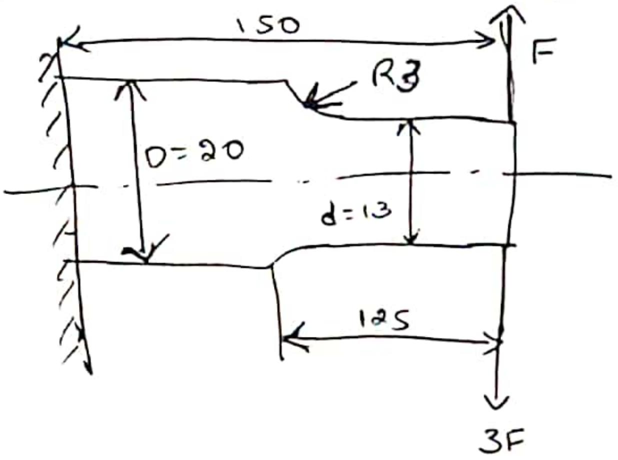
$$\frac{1 \times \left(\frac{50.9296 \times 10^3}{d^2}\right)}{0.75 \times 0.85 \times 0.91 \times 427.6} + \frac{\left(\frac{101.8591 \times 10^3}{d^2}\right)}{689.4} = \frac{1}{2}$$

$$\frac{205.2104}{d^2} + \frac{147.7503}{d^2} = \frac{1}{2}$$

$$\boxed{d = 26.5729\ mm}$$

9

7) A cantilever beam made of cold drawn carbon steel ($\sigma_u = 550 MPa$, $\sigma_y = 470 MPa$ & $\sigma_{en} = 275 MPa$) of circular crossection is subjected to a load ~~with vari~~ which varies from $-F$ to $+3F$ determine the max. load that the member can with stand for infinite life using the factor of safety as 2.



soln:- $\sigma_u = 550 MPa$, $n = 2$

$\sigma_y = 470 MPa$

$\sigma_{en} = 275 MPa$

min. load $= -F$

max. load $= +3F$

Since the given material is ductile, the design equation to be used is soderberg eqn.

$$\frac{K_{ef} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \rightsquigarrow [\text{eqn } 2.21(c) \text{ Pg } 25]$$

Since the possibility of failure is across the fillet

max. Bending moment at fillet, $M_{max} = F_{max} \times l$

$= 3F \times 125$

$\boxed{M_{max} = 375 F}$

min. Bending moment at fillet, $M_{min} = F_{min} \times l$

$= -F \times 125$

$\boxed{M_{min} = -125 F}$

max. Bending stress, $\sigma_{max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 375 F}{\pi (13)^3}$

$\therefore \sigma_{bmax} = 1.7386 F$

~~Ela~~ min. Bending stress, $\sigma_{bmin} = \frac{32 M_{min}}{\pi d^3}$

$$= \frac{32 \times -125F}{\pi \times (13)^3}$$

$$\boxed{\sigma_{bmin} = -0.5795F}$$

mean ~~Bending~~ stress $= \sigma_m = \frac{\sigma_{bmax} + \sigma_{bmin}}{2}$

$$= \frac{1.7386F - 0.5795F}{2}$$

$$\boxed{\sigma_m = 0.57955F}$$

Amplitude stress

$$\sigma_a = \frac{\sigma_{bmax} - \sigma_{bmin}}{2} = \frac{1.7386F + 0.5795F}{2}$$

$$\boxed{\sigma_a = 1.15905F}$$

To find K_{tf}

$K_{tf} = 1 + q(K_t - 1)$

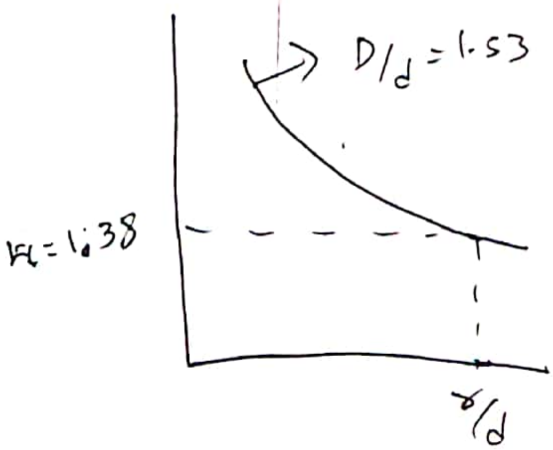
$q = 1$ (assume)

To find K_t

From Fig 2.25 Pg 43 (circular shaft)

$\frac{r}{d} = \frac{3}{13} = 0.23$

$\frac{D}{d} = \frac{20}{13} = 1.53$



$$K_{tf} = 1 + q(K_t - 1)$$

$$= 1 + 1(1.38 - 1)$$

$$\boxed{K_{tf} = 1.38}$$

A = 1 (reverse Bending)

B = 0.8 (assume)

c = 0.8 (assume)

$$\frac{K_{tf} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1.38 \times 1.15905F}{1 \times 0.8 \times 0.8 \times 275} + \frac{0.57955F}{470} = \frac{1}{2}$$

$$10^{-3} \times 9.08800 F + 1.2330 \times 10^{-3} F = \frac{1}{2}$$

$$\boxed{F = 48.44 N}$$

$$F_{max} = 3F = 3 \times F$$

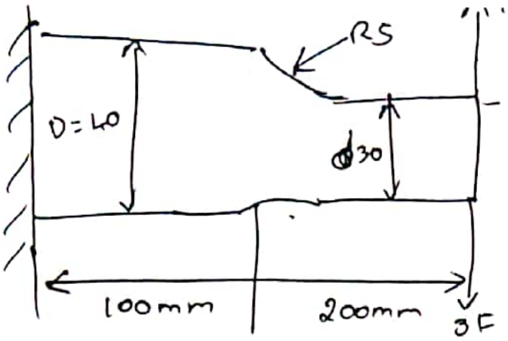
$$F_{max} = 3 \times 48.44$$

$$\boxed{F_{max} = 145.32 N}$$

exam

7) A cantilever beam shown in figure is subjected to load variation from $-F$ to $3F$ determine the max load that the member can withstand for an infinite life using a Factor of safety of 2. The material of the beam is SAE 1025 water quanched steel.

$(\sigma_u = 620.8 \text{ MPa}, \ \sigma_y = 400.1 \text{ MPa} \ \& \ \sigma_{en} = 345.2 \text{ MPa})$.



Soln

$\sigma_u = 620.8 \, MPa$

$\sigma_y = 400.1 \, MPa$

$\sigma_{en} = 345.2 \, MPa$

min. load = $-F$

max. load = $3F$

$D = 40 \, mm$

$d = 30 \, mm$

$r = 5 \, mm$

$n = 2$

WKT

$$\frac{K_{tf} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \rightsquigarrow [\text{eq}^n \ 2.21(c) \ Pg \ 25]$$

max. Bending moment at fillet, $M_{max} = 3F \times 200$

$= \underline{600F}$

min Bending moment at fillet, $M_{min} = -F \times 200$

$= \underline{-200F}$

max. Bending stress $\sigma_{max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 600F}{\pi \times (30^3)}$

$\sigma_{max} = 0.2263F$

min. Bending stress, $\sigma_{min} = \frac{32 M_{min}}{\pi d^3} = \frac{32 \times -200F}{\pi \times (30^3)}$

$$\boxed{\sigma_{min} = -0.0754F}$$

Mean stress = $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{0.2263F - 0.0754F}{2}$

$$\therefore \boxed{\sigma_m = 0.07545F}$$

Amplitude stress = $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{0.2263F + 0.0754F}{2}$

$$\boxed{\sigma_a = 0.15085F}$$

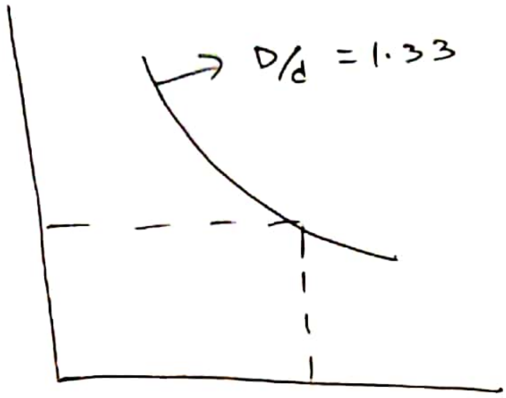
To find K_{tf}.
wkt

$K_{tf} = 1 + q(K_t - 1)$

A: 1 (bending) B = 0.8 (assume)
C = 0.8 (assume)
q = 1 (assume)

To find K_t

From Fig 2.25 Pg 43



$\frac{r}{d} = \frac{5}{30} = 0.16$

$\frac{D}{d} = \frac{40}{30} = 1.33$

$K_{ft} = 1 + q(K_t - 1)$
$= 1 + 1(1.47 - 1)$

$\boxed{K_{ft} = 1.47}$

$$\frac{1.47 \times 0.15085F}{1 \times 0.8 \times 0.8 \times 345.2} + \frac{0.07545F}{400.1} = \frac{1}{2}$$

$\boxed{F = 419.358\,N}$

$F_{max} = 3F = 419.358 \times 3$

$\therefore \boxed{F_{max} = 1258.076\,N}$

20/09/18

axial load → Axial stress / Direct Stress / normal Stress

Tensile, compressive

$$\sigma_d = P/A$$

For shaft/circular
$$A = \frac{\pi d^2}{4}$$

For Rectangular
$$A = b \times h$$

2) Bending load

$$\sigma_b = \frac{M \times I}{y}$$

For circular
$$\sigma = \frac{32M}{\pi d^3}$$

3) Torsional load / twisting :- shear stress

For circular
$$\tau = \frac{16T}{\pi d^3}$$

$$\therefore \tau = \frac{T \times r}{J}$$

The load act on the Top of the bar (or) shaft is know Bending load.

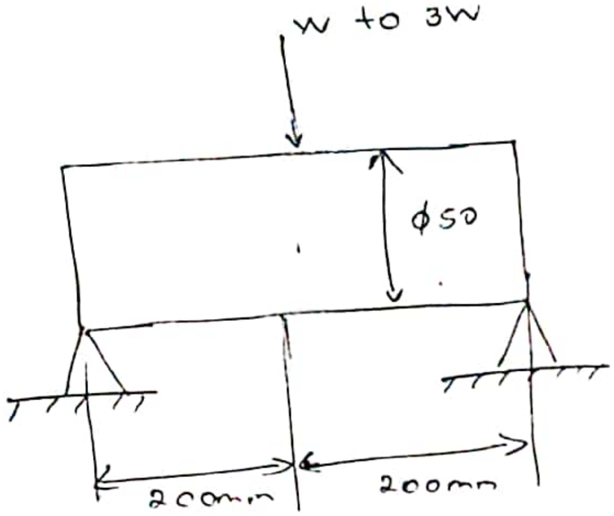
The load act on the axis of shaft (or) object is known as axial load.

(Impact loading axial & Bending problem)

deflection → Bending

extension → axial

87) Determine the max. load for a simply supported beam as shown in figure. take $\sigma_u = 700$ MPa, $\sigma_y = 520$ MPa & $\sigma_{en} = 320$ MPa. Take $C = 0.9$, $B = 0.85$ & factor of safety 1.25.



Soln:- $\sigma_u = 700$ MPa

$\sigma_y = 520$ MPa

$\sigma_{en} = 320$ MPa

$n = 1.25$

$C = 0.9$

$B = 0.85$

min. load, $P_{min} = W$

max. load $P_{max} = 3W$

Soderberg eqn

$$\frac{K_f \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

For a simply supported beam carrying load at its centre.
From due table 1.4 Pg 15

→ max. bending moment $= \frac{WL}{4}$

max Bending moment $= \frac{WL}{4}$

$$= \frac{3W \times 400}{4}$$

$$\boxed{M_{max} = 300W}$$

max. bending stress

$$\sigma_{max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 300W}{\pi \times (50^3)}$$

$$\boxed{\sigma_{max} = 0.0244\,W}$$

min. Bending moment $= \frac{WL}{4}$

$$= \frac{W \times 400}{4}$$

$$\boxed{M_{min} = 100W}$$

Min. bending stress

$$\sigma_{min} = \frac{32 M_{min}}{\pi d^3} = \frac{32 \times 100W}{\pi \times (50^3)}$$

$$\boxed{\sigma_{min} = 8.1487 \times 10^{-3} W}$$

mean stress $= \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$$\sigma_m = \frac{0.0244W + 8.1487 \times 10^{-3} W}{2}$$

$$\boxed{\sigma_m = 0.0162\,W}$$

amplitude stress, $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

$$\sigma_a = \frac{0.0244W - 8.1487 \times 10^{-3} W}{2}$$

$$\boxed{\sigma_a = 8.12565 \times 10^{-3} W}$$

Scanned with CamScanner

To find the K_{tf}

$K_{tf} = 1 + q(K_t - 1)$, ~~q=1 (assume)~~

~~To find the K_t~~

$\therefore K_{tf} = 1$ (assume)

In this type of problem ie when a stress conc is not discussed then take K_{tf} value as 1

The correction factor

$A = 1$ (Bending)

$B = 0.85$

$C = 0.9$

substitute all the values in sodenberg equation.

$$\frac{1 \times 8.1256 \times 10^{-3} W}{1 \times 0.85 \times 0.9 \times 320} + \frac{0.0162 W}{520} = \frac{1}{1.25}$$

(or)

$\boxed{W = 12432.658 \text{ N}} \boxed{= 12.4326 \text{ kN}}$

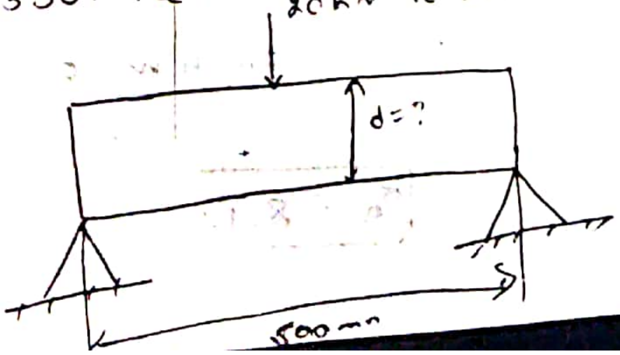
$W = 3W = 3 \times 12432.658 \text{ N}$

(or)

$\boxed{W = 37,297.975 \text{ N}} \boxed{= 37.297 \text{ kN}}$

q) A circular bar of 500mm length is supported freely at its two end it is acted upon by an central concentrated cyclic load. having a minimum value of 20kN & a max. value of 50kN. Determine the diameter of the bar by taking FOS = 2. size effect of 0.85, surface finish factor of 0.9. the material properties of the bar is given by $\sigma_u = 650$ MPa, $\sigma_y = 500$ MPa & $\sigma_{en} = 350$ MPa.

Sol^n



$L = 500 mm$

$P_{min} = 20 kN$

$P_{max} = 50 kN$

$d = ?$

$n = 2$

$B = 0.85$

$C = 0.9$

$\sigma_u = 650 MPa$

$\sigma_y = 500 MPa$

$\sigma_{en} = 350 MPa$

soderberg eq^n

$$\frac{K_{ff} \times \sigma_a}{ABC \times \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$ → [eq^n 2.21 (c), Pg 2.]

From the table 1.4 Pg 15

max. Bending moment $= \frac{WL}{4} = \frac{PL}{4}$

max. Bending moment $= \frac{P_{max} L}{4}$

$= \frac{50 \times 10^3 \times 500}{4}$

$\therefore \boxed{M_{max} = 6.25 \times 10^6 N\text{-}mm}$

min Bending moment $= \frac{P_{min} \times L}{4} = \frac{20 \times 10^3 \times 500}{4} = 2.5 \times 10^6 N\text{-}mm$

For circular

max. Bending stress $= \sigma_{max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 6.25 \times 10^6}{\pi \times d^3}$

$\therefore \boxed{\sigma_{max} = \frac{63.66197 \times 10^6}{d^3}}$

min. bending stress $= \sigma_{min} = \frac{32 M_{min}}{\pi d^3} = \frac{32 \times 2.5 \times 10^6}{\pi \times d^3}$

$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$\therefore \boxed{\sigma_{min} = \frac{25.464 \times 10^6}{d^3}}$

In this type of problem i.e when a stress conc is not discussed then take the K_{ff} value as 1

i.e $\therefore \boxed{K_{ff} = 1}$

Correction factor

$A = 1$ (Bending)

$B = 0.85$

$C = 0.9$

$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

$\boxed{\sigma_a = \frac{19.09895 \times 10^6}{d^3}}$

$\boxed{\sigma_m = \frac{44.5624 \times 10^6}{}}$

substitute all the values in Soderberg eq^n

$$\frac{1 \times 19.09895 \times 10^6}{1 \times 0.85 \times 0.9 \times 350 d^3} + \frac{44.5629 \times 10^6}{d^3 (500)} = \frac{1}{2}$$

$\boxed{d = 68.4641 mm}$

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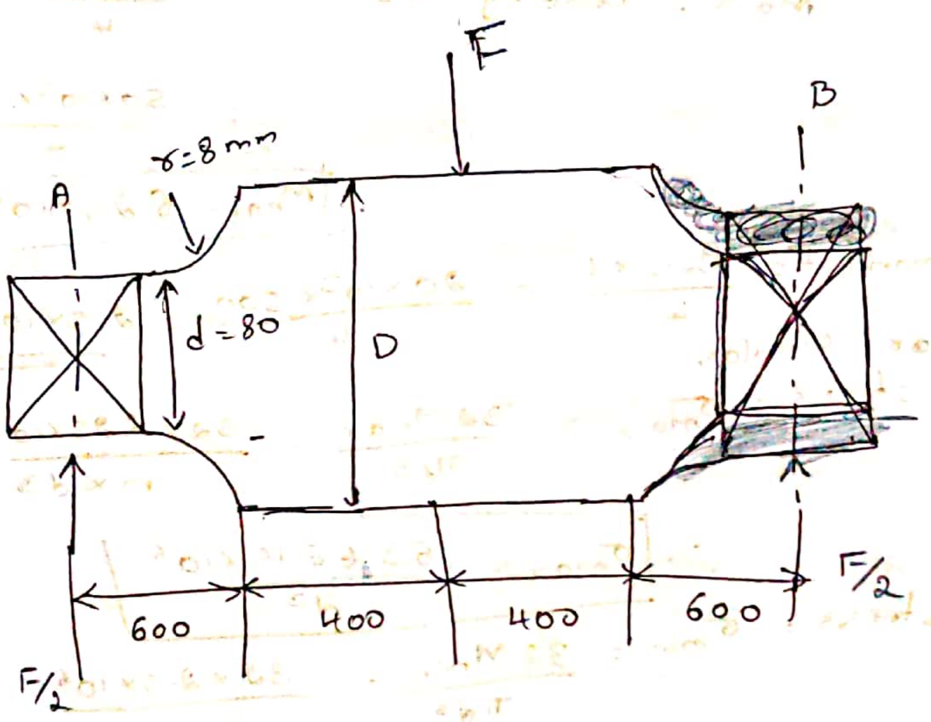
10) A non-rotating shaft is shown in figure. it is subject to a load F varying from 4000 to 12,000 N. The shaft material has $\sigma_u = 600 MPa$, $\sigma_{en} = 300 MPa$. The correction co-efficient for surface, size & load are 0.8, 0.85 & 0.9 respectively. Find the dimension 'D' for a FoS of 3.5 & notch sensitivity Index. $q = 0.9$.



solh

$F_{min} = 4000 N$	$A = 0.9$	$n = 3.5$
$F_{max} = 12,000 N$	$B = 0.85$	$r = 8mm$
$\sigma_u = 600 MPa$	$C = 0.8$	$d = 80mm$
$\sigma_{en} = 300 MPa$	$q = 0.9$	$D = ?$

since the distance are symmetrical the reaction at the bearings are F/2.

The possibility of failure is cross the fillet

i.e, Bending moment across fillet = $F/2 \times 600$

max. Bending moment, $M_{max} = \frac{F_{max}}{2} \times 600$

$= \frac{12,000 \times 600}{2}$

$\therefore \boxed{M_{max} = 3.6 \times 10^6 N\text{-}mm}$

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min. Bending moment, $M_{min} = \frac{F_{min}}{2} \times 600$

$= \frac{4000 \times 600}{2}$

$$\boxed{M_{min} = 1.2 \times 10^6 \ N\text{-}mm}$$

max. bending stress,

$$\sigma_{max} = \frac{32 \times M_{max}}{\pi d^3} = \frac{32 \times 3.6 \times 10^6}{\pi \times (80^3)}$$

$$\boxed{\sigma_{max} = 71.6197 \ N/mm^2}$$

min. bending stress, $\sigma_{min} = \frac{32 \times M_{min}}{\pi d^3} = \frac{32 \times 1.2 \times 10^6}{\pi \times (80^3)}$

$$\boxed{\sigma_{min} = 23.8732 \ N/mm^2}$$

mean stress, $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{71.6197 + 23.8732}{2}$

$$\therefore \boxed{\sigma_m = 47.7464 \ N/mm^2}$$

amplitude stress, $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{71.6197 - 23.8732}{2}$

$$\therefore \boxed{\sigma_a = 23.8732 \ N/mm^2}$$

The design equation to be used is Goodman's equation (ductile material)

$$\frac{K_{f} \sigma_a}{ABC \ \sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n} \rightarrow [\text{eq}^n \ 2.21(a), \ Pg \ 25]$$

$$\sigma_a K_f = \left(\frac{1}{n} - \frac{\sigma_m}{\sigma_u}\right)(ABC \times \sigma_{en})$$

$$23.8732 \ K_f = \left(\frac{1}{3.5} - \frac{47.7464}{600}\right)(0.9 \times 0.85 \times 0.8 \times 300)$$

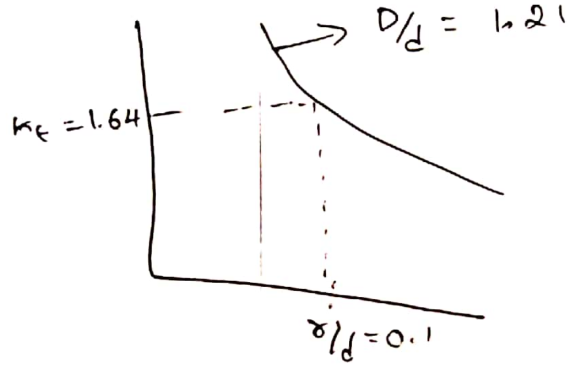
$$\boxed{k_{tf} = 1.58}$$

we know that $k_{tf} = 1 + q(k_t - 1)$

$$1.58 = 1 + 0.9(k_t - 1)$$

$$\boxed{k_t = 1.64}$$

From: Fig 2.25 pg 43



$$\frac{r}{d} = \frac{8}{80} = 0.1$$

$$\frac{D}{d} = 1.21$$

$$D = 1.21 \times d$$

$$D = 1.21 \times 80$$

$$\boxed{D = 96.8 \text{ mm}}$$

Design for dynamic strength under combined loading condition

1) equivalent normal stress

$$\sigma_{eq-n} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right) \frac{k_{tf}\,\sigma_a}{ABC}$$ → [eq^n 2.22(a), Pg 26]

2) equivalent shear stress

$$\tau_{ea} = \tau_m + \left(\frac{\tau_y}{\sigma_{en}}\right) \frac{K_{sf}\,\tau_a}{ABC}$$ → [eq^n 2.22(b), Pg 26]

3) The equivalent maximum shear stress

$$\tau_{eq}(max) = \sqrt{(\tfrac{1}{2}\sigma_{eq-n})^2 + (\tau_{eq})^2} \Big\} \rightarrow [eq^n\, 2.23(a),\ Pg\ 26]$$
$$= \tau_y/n$$

4) The equivalent max. normal stress to be used when designing with brittle materials.

$$\sigma_{eq-n}(max) = \frac{1}{2}\cdot\sigma_{eq-n} + \sqrt{(\tfrac{1}{2}\sigma_{eq-n})^2 + (\tau_{eq})^2} \Big\} \rightarrow [eq^n\, 2.23(, Pg\ 26$$
$$= \sigma_y/n$$

9/10/18

Problems

1) A cold drawn ~~steel~~ steel rod of circular cross-section is subjected to a variable bending momentum of 540N-m to 1080 N-m. As the axial load varies from 4kN to 10kN. The maximum axial load & maximum bending moment occurs at the same Instant, the material has a ultimate stress of 570 MPa & yield stress of 420MPa. Determine the diameter of the rod taking factor of safety as 2.

Soln

$M_{min} = 540\times10^3$ N-mm

$M_{max} = 1080\times10^3$ N-mm

$P_{max} = 10\times10^3$ N

$P_{min} = 4\times10^3$ N

$\sigma_u = 570$ MPa

$\sigma_y = 420$ MPa

$d = ?$

$n = 2$

For steel

$\sigma_{en} = 0.5\,\sigma_u$

$= 0.5\times570$

$\boxed{\sigma_{en} = 285\,MPa}$

Considering bending load

$$\sigma_{bmax} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 1080 \times 10^3}{\pi d^3}$$

$$\therefore \boxed{\sigma_{bmax} = \frac{11 \times 10^6}{d^3}}$$

$$\sigma_{bmin} = \frac{32 M_{min}}{\pi d^3} = \frac{32 \times 540 \times 10^3}{\pi d^3}$$

$$\boxed{\sigma_{bmin} = \frac{5.5003 \times 10^6}{d^3}}$$

mean stress $(\sigma_m) = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{\left(\frac{11 \times 10^6}{d^3}\right) + \left(\frac{5.5003 \times 10^6}{d^3}\right)}{2}$

$$\boxed{\sigma_m = \frac{8.25 \times 10^6}{d^3}}$$

Amplitude stress (σ_a)

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\left(\frac{11 \times 10^6}{d^3}\right) - \left(\frac{5.5003 \times 10^6}{d^3}\right)}{2}$$

$$\boxed{\sigma_a = \frac{2.7498 \times 10^6}{d^3}}$$

Equivalent normal stress

$(\sigma_{eq-n}) = \sigma_m + \left(\dfrac{\sigma_y}{\sigma_{en}}\right)\left(\dfrac{k_{tf}\,\sigma_a}{ABC}\right) \longrightarrow$ [eqn 2.22(a), Pg 26]

$$(\sigma_{eq-n})_B = \left(\frac{8.25 \times 10^6}{d^3}\right) + \left(\frac{420}{285}\right)\frac{1 \times \left(\frac{2.75 \times 10^6}{d^3}\right)}{1 \times 0.8 \times 0.8}$$

$$\boxed{(\sigma_{eq-n})_B = \frac{14.5822 \times 10^6}{d^3}} \longrightarrow (1)$$

Assume
$k_{tf} = 1$
$A = 1.0$ (For Reverse bending)
$B = 0.8$
$C = 0.8$

consider axial load

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{10\times10^3\times4}{\pi d^2}$$

$$\boxed{\sigma_{max} = \frac{12.7323\times10^3}{d^2}}$$

$$\sigma_{min} = \frac{P_{min}}{A} = \frac{4\times10^3\times4}{\pi d^2}$$

$$\boxed{\sigma_{min} = \frac{5.0929\times10^3}{d^2}}$$

mean stress $(\sigma_m) = \dfrac{\sigma_{max}+\sigma_{min}}{2} = \dfrac{\left(\frac{12.7323\times10^3}{d^2}\right)+\left(\frac{5.0929\times10^3}{d^2}\right)}{2}$

$$\therefore \boxed{\sigma_m = \frac{8.9126\times10^3}{d^2}}$$

Amplitude stress (σ_a)

$$\sigma_a = \frac{\sigma_{max}-\sigma_{min}}{2} = \frac{\left(\frac{12.7323\times10^3}{d^2}\right)-\left(\frac{5.0929\times10^3}{d^2}\right)}{2}$$

$$\boxed{\sigma_a = \frac{3.8197\times10^3}{d^2}}$$

Equivalent normal stress

$$(\sigma_{eq-n})_{Axial} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right)\left(\frac{K_{tf}\,\sigma_a}{ABC}\right) \rightsquigarrow (\text{Eq } 2.22(a),\ \text{Pg } 26)$$

$$(\sigma_{eq-n})_A = \left(\frac{8.9126\times10^3}{d^2}\right) + \left(\frac{420}{285}\right)\left(\frac{1\times\left(\frac{3.8197\times10^3}{d^2}\right)}{0.7\times0.8\times0.8}\right)$$

$$\boxed{(\sigma_{eq-n})_A = \frac{21.4774\times10^3}{d^2}} \rightsquigarrow (2)$$

Assume
$K_{tf} = 1$
$A = 0.7$ (axial load)
$B = 0.8$
$C = 0.8$

$$(\sigma_{eq\cdot n})_{max} = (\sigma_{eq\cdot n})_B + (\sigma_{eq\cdot n})_A = \sigma_y/n$$

$$\frac{420}{2} = \frac{14.5822\times10^6}{d^3} + \left(\frac{21.4774\times10^3}{d^2}\right)$$

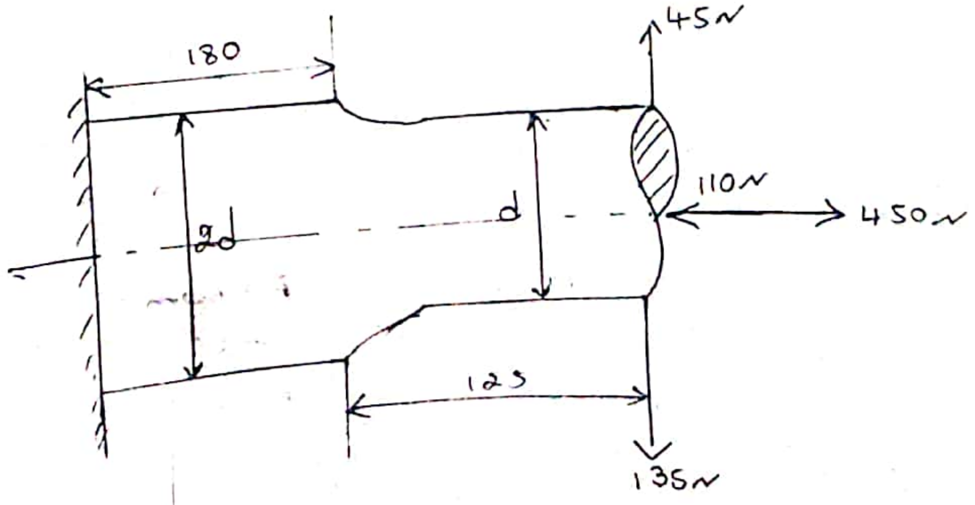
$$210 = \frac{1}{d^3}\left[14.5822\times10^6 + 21.4774\times10^3 d\right]$$

$$210 d^3 = 14.5822\times10^6 + 21.4774\times10^3 d$$

$$210 d^3 - 21.4774\times10^3 d - 14.5822\times10^6 = 0$$

$$\boxed{d = 41.93\,mm} \approx \underline{42mm}$$

2) A cantilever beam made of steel has ultimate tensile strength of 560 MPa, yield strength of 470 MPa & Endurance limit in reverse bending of 275 MPa is loaded as shown in figure. Determine the required diameter at the change of section for an infinite life using a FOS of 2. The theoretical stress conc for bending & axial conditions are 1.44 & 1.63 respectively. The notch sensitivity index can be taken as 0.88.



Sol:

$F_{bmax} = 135N$ (②)

$F_{bmin} = -45N$ (③)

$P_{max} = 450N$ ($\because$ tensile)

$P_{min} = -110N$ ($\because$ compressive)

$\sigma_u = 560MPa$

$\sigma_{en} = 275MPa$

$q = 0.88$

$\sigma_y = 470MPa$

$n = 2.$

stress conc (k_t)

$(k_t)_B = 1.44$

$(k_t)_A = 1.63$

consider bending load:-

Failure of component happens across the fillet, $M = F \times l$

max. Bending moment $= (135 \times 125) = \underline{16.875 \times 10^3 N\text{-}mm}$

min. Bending moment, $M_{min} = (-45 \times 125) = \underline{-5.625 \times 10^3 N\text{-}mm}$

Considering Axial load

$P_{max} = 450N$ (tensile)

$P_{min} = -110N$

~~$\sigma_u = 56MP$~~

consider the bending load

max. bending stress, $\sigma_{max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 16.875 \times 10^3}{\pi d^3}$

$$\therefore \boxed{\sigma_{max} = \frac{171.8873 \times 10^3}{d^3}}$$

min. bending stress, $\sigma_{min} = \frac{32 M_{min}}{\pi d^3} = \frac{-32 \times 5.625 \times 10^3}{\pi d^3}$

$$\therefore \boxed{\sigma_{min} = -\frac{57.2957 \times 10^3}{d^3}}$$

mean stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{171.8873 \times 10^3 - 57.2957 \times 10^3}{2 d^3}$$

$$\boxed{\sigma_m = \frac{57.2958 \times 10^3}{d^3}}$$

Amplitude stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$= \frac{171.5873 \times 10^3 + 57.2957 \times 10^3}{2d^3}$$

$$\boxed{\sigma_a = \frac{114.5915 \times 10^3}{d^3}}$$

Equivalent normal stress

$$\sigma_{eq-n} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right) \frac{K_{ff}\,\sigma_a}{ABC} \rightarrow (eq^n\ 2.22(a),\ Pg\ 26)$$

$A = 1$ (Bending)

$B = 0.8$

$C = 0.8$

To find K_{ff}

$$K_{ff} = 1 + q(K_f - 1)$$

$$= 1 + 0.88(1.44 - 1)$$

$$\boxed{K_{ff} = 1.3872}$$

$$(\sigma_{ea-n})_B = \left(\frac{57.2958 \times 10^3}{d^3}\right) + \left(\frac{470}{275}\right)\left(\frac{1.3872 \times 114.5915 \times 10^3}{d^3 \times 1 \times 0.8 \times 0.8}\right)$$

$$\boxed{(\sigma_{eq-n})_B = \frac{481.7948 \times 10^3}{d^3}} \rightarrow 1$$

Consider axial load

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{450 \times 4}{\pi d^2}$$

$$\boxed{\sigma_{max} = \frac{572.9577}{d^2}}$$

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$$\sigma_{min} = \frac{P_{min}}{A} = \frac{-110 \times 4}{\pi d^2}$$

$$\boxed{\sigma_{min} = \frac{-140.0563}{d^2}}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{572.9577 - 140.0563}{2 d^2}$$

$$\boxed{\sigma_m = \frac{216.4507}{d^2}}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{572.9577 + 140.0563}{2 d^2}$$

$$\boxed{\sigma_a = \frac{356.507}{d^2}}$$

$$(\sigma_{eq-n})_A = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right) \frac{K_{tf}\, \sigma_a}{ABC}$$

To find K_{tf}

$K_{tf} = 1 + q(K_t - 1)$

$= 1 + 0.88(1.63 - 1)$

$\boxed{K_{tf} = 1.5544}$

$A = 0.7$ (axial load)

$B = 0.8$

$C = 0.8$

$$(\sigma_{eq-n})_A = \left(\frac{216.4507}{d^2}\right) + \left(\frac{470}{275}\right)\left(\frac{1.5544 \times 356.507}{d^2 \times 0.7 \times 0.8 \times 0.8}\right)$$

$$\boxed{(\sigma_{eq-n})_A = \frac{2.3305 \times 10^3}{d^2}} \rightsquigarrow 2$$

$(\sigma_{eq-n})_{max} = (\sigma_{eq-n})_B + (\sigma_{eq-n})_A = \sigma_y/n$

$$\frac{470}{2} = \left(\frac{481.7948\times10^3}{d^3}\right) + \left(\frac{2.3305\times10^3}{d^2}\right)$$

$$\frac{470}{2} = \frac{1}{d^3}\left[481.7948\times10^3 + 2.3305\times10^3 d\right]$$

$$235d^3 - 2.3305\times10^3 d - 481.7948\times10^3 = 0$$

$$\boxed{d = 12.96\,mm}$$

3) A transmission shaft carries a pulley midway b/w two bearings. The bending moment at the pulley varies from 200N-m to 600N-m. As the torsional moment of shaft varies from 70N-m to 200N-m. The frequency of variation of bending & torsional moment are equal to the speed of the shaft. The shaft is made of steel FiE 400. ($\sigma_u = 540MPa$, $\sigma_y = 400MPa$) The corrected endurance strength of the shaft is 200MPa. Determine the diameter of the shaft by taking an FOS of 2.

Sol^n :- $M_{min} = 200N\text{-}m$ | $T_{min} = 70N\text{-}m$

$M_{max} = 600N\text{-}m$ | $T_{max} = 200N\text{-}m$

$\sigma_u = 540MPa$

$\sigma_y = 400MPa$

$\sigma_{en} = 200MPa$

$n = 2$

$d = ?$

considering the ~~Stress~~ moment

$$\sigma_{b\,min} = \frac{32 M_{min}}{\pi d^3} = \frac{32 \times 200 \times 10^3}{\pi \times d^3} = \frac{2.0371 \times 10^6}{d^3}$$

$$\sigma_{b\,max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 600 \times 10^3}{\pi \times d^3} = \frac{6.1115 \times 10^6}{d^3}$$

mean stress, $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$

$$= \frac{6.1115 \times 10^6 + 2.0371 \times 10^6}{2 d^3}$$

$$\boxed{\sigma_m = \frac{4.0743 \times 10^6}{d^3}}$$

Amplitude stress, $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

$$= \frac{6.1115 \times 10^6 - 2.0371 \times 10^6}{2 d^3}$$

$$\boxed{\sigma_a = \frac{2.03725 \times 10^6}{d^3}}$$

$$(\sigma_{eq-n})_B = \sigma_m + \left(\frac{\sigma_{yp}}{\sigma_{en}}\right) \frac{K_{ff}\,\sigma_a}{ABC} \rightsquigarrow \text{eq}^n\ 2.22(a),\ \text{Pg } 26$$

To find $K_{ff} = 1$ (assume)

$A = 1.0$ (for bending)

$B = 0.8$

$C = 0.8$

$$(\sigma_{eq\cdot n})_B = \frac{4.0743 \times 10^6}{d^3} + \left(\frac{400}{200}\right) \frac{1 \times 2.03725 \times 10^6}{d^3 \times 1 \times 0.8 \times 0.8}$$

$$\boxed{(\sigma_{eq-n})_B = \frac{10.4407 \times 10^6}{d^3}} \rightsquigarrow (1)$$

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considering a torsional moment

$$\tau_{max} = \frac{16\,T_{max}}{\pi d^3} = \frac{16 \times 200 \times 10^3}{\pi d^3} = \frac{1.0185 \times 10^6}{d^3}$$

$$\tau_{min} = \frac{16\,T_{min}}{\pi d^3} = \frac{16 \times 70 \times 10^3}{\pi d^3} = \frac{356.5070 \times 10^3}{d^3}$$

$$\tau_{mean} = \frac{\tau_{max} + \tau_{min}}{2}$$

$$= \frac{356.5070 \times 10^3 + 1.0185 \times 10^6}{2 d^3}$$

$$\boxed{\tau_{mean} = \frac{687.5035 \times 10^3}{d^3}}$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{1.0185 \times 10^6 - 356.5070 \times 10^3}{2 d^3}$$

$$\boxed{\tau_a = \frac{330.9965 \times 10^3}{d^3}}$$

considering a equivalent shear stress

$$\tau_{eq} = \tau_m + \left(\frac{\tau_y}{\sigma_{en}}\right) \frac{K_{sf}\,\tau_a}{ABC}$$

WKT $\quad \tau_y = \sigma_y/2 = \frac{400}{2}$

$$\boxed{\tau_y = 200\,MPa}$$

$K_{sf} = 1$

$A = 0.5$ (torsion)

$B = 0.8$

$C = 0.8$

$$\tau_{eq} = \frac{687.5035\times10^3}{d^3} + \left(\frac{200}{200}\right)\left(\frac{1\times 330.4965\times10^3}{d^3\times0.5\times0.8\times0.8}\right)$$

$$\boxed{\tau_{eq} = \frac{1.72186\times10^6}{d^3}}$$

max. equivalent shear stress

$$(\tau_{eq-n})_{max} = \sqrt{(\tfrac{1}{2}\sigma_{eq-n})^2+(\tau_{eq})^2} = \tau_y/n \rightsquigarrow eq^n\ 2.23(a)$$

Pg 26

$$\boxed{\tau_y/n = \frac{200}{2} = 100}$$

$$\frac{200}{2} = \sqrt{\left(\frac{10.4407\times10^6}{2d^3}\right)^2+\left(\frac{1.72186\times10^6}{d^3}\right)^2}$$

$$100 = \frac{1}{d^3}\sqrt{(5.22035\times10^6)^2+(1.72186\times10^6)^2}$$

$$100 = \frac{5.4969\times10^6}{d^3}$$

$$d^3 = \frac{5.4969\times10^6}{100}$$

$$\boxed{d = 38.0225\,mm}$$

maximum equivalent normal stress

$$(\sigma_{eq-n})_{max} = \frac{1}{2}\sigma_{eq-n} + \sqrt{(\tfrac{1}{2}\sigma_{eq-n})^2+(\tau_{eq})^2} = \frac{\sigma_y}{n} \rightsquigarrow$$

$eq^n\ 2.23(b)$, Pg 26

$$\frac{400}{2} = \frac{1}{2}\left(\frac{10.4407\times10^6}{d^3}\right) + \sqrt{\left(\frac{10.4407\times10^6}{2d^3}\right)^2+\left(\frac{1.72186\times10^6}{d^3}\right)^2}$$

$$200 = \frac{5.22035\times10^6}{d^3} + \frac{1}{d^3}\left[5.4969\times10^6\right]$$

$$200 = \frac{10.71725 \times 10^6}{d^3}$$

$$\boxed{d = 37.7008 mm}$$

∴ The minimum diameter of the shaft is ∴ 38.01 mm (adopt the bigger value)

Imp

4) A hot rolled shaft is subjected to a torsional load that varies from 330 N-m clock wise to 110 N-m counter clock wise (Anti clock wise). An apply bending moment at the critical section varies from 440 N-m to -220 N-m. The shaft is of uniform cross section & no key way present at critical section. Determine the required shaft diameter the material has an ultimate stress of 550 MPa & yield strength of 410 MN/m² take edurance limit as of the (½) half the ultimate strength. FOS = 1.5 size & surface correction co-efficient as 0.85 & 0.62 respectively.

Soln

$T_{max} = 330 N\text{-}m$ (C)

$T_{min} = -110 N\text{-}m$ (C)

$M_{max} = 440 N\text{-}m$

$M_{min} = -220 N\text{-}m$

$d = ?$

$\sigma_u = 550 MPa$

$\sigma_y = 410 MPa$

$\sigma_{en} = 0.5 \times \sigma_u$

$= 0.5 \times 550$

$$\boxed{\sigma_{en} = 275 MPa}$$

$n = 1.5$

$B = 0.85$

$C = 0.62$

$d = ?$

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Consider a torsional moment

$$\tau_{max} = \frac{16\,T_{max}}{\pi d^3} = \frac{16 \times 330 \times 10^3}{\pi \times d^3} = \frac{1.6806 \times 10^6}{d^3}$$

$$\tau_{min} = \frac{16\,T_{min}}{\pi d^3} = \frac{-16 \times 110 \times 10^3}{\pi d^3} = -\frac{560.2253 \times 10^3}{d^3}$$

$$\tau_{mean} = \frac{\tau_{max} + \tau_{min}}{2} = \frac{1.6806 \times 10^6 - 560.2253 \times 10^3}{2 d^3}$$

$$\boxed{\tau_m = \frac{560.18735 \times 10^3}{d^3}}$$

$$\tau_{amplitude} = \frac{\tau_{max} - \tau_{min}}{2} = \frac{1.6806 \times 10^6 + 560.2253 \times 10^3}{2 d^3}$$

$$\boxed{\tau_a = \frac{1.1204 \times 10^6}{d^3}}$$

equivalent shear stress.

$$\tau_{eq} = \tau_m + \left(\frac{\tau_y}{\sigma_{en}}\right) \frac{k_{sf}\,\tau_a}{ABC}$$ → eq^n 8.22(b), Pg 26

$k_{sf} = 1$ A = 0.5

$\tau_y = \frac{\sigma_y}{2} = \frac{410}{2}$ B = 0.85

C = 0.62

$$\boxed{\tau_y = 205\ MPa}$$

$$\tau_{eq} = \frac{560.18735 \times 10^3}{d^3} + \left(\frac{205}{275}\right)\left(\frac{1 \times 1.1204 \times 10^6}{d^3 \times 0.5 \times 0.85 \times 0.62}\right)$$

$$\boxed{\tau_{eq} = \frac{3.7298 \times 10^6}{d^3}}$$

consider a bending moment

$$\sigma_{max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 440 \times 10^3}{\pi d^3} = \frac{4.4818 \times 10^6}{d^3}$$

$$\sigma_{min} = \frac{32 M_{min}}{\pi d^3} = \frac{32 \times -220 \times 10^3}{\pi d^3} = \frac{-2.2409 \times 10^6}{d^3}$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{4.4818 \times 10^6 - 2.2409 \times 10^6}{2 d^3}$$

$$\boxed{\sigma_m = \frac{1.12045 \times 10^6}{d^3}}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{4.4818 \times 10^6 + 2.2409 \times 10^6}{2 d^3}$$

$$\boxed{\sigma_a = \frac{3.36135 \times 10^6}{d^3}}$$

equivalent normal stress

$$(\sigma_{eq-n})_B = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right) \frac{K_{tf} \times \sigma_a}{ABC} \rightarrow [\text{eq}^n\ 2.22(a), \text{P.g } 26]$$

$K_{tf} = 1$

$A = 1$

$B = 0.85$

$C = 0.62$

$$= \frac{1.12045 \times 10^6}{d^3} + \left(\frac{410}{275}\right)\left(\frac{1 \times 3.36135 \times 10^6}{d^3 \times 1 \times 0.85 \times 0.62}\right)$$

$$\boxed{(\sigma_{eq-n})_B = \frac{10.5998 \times 10^6}{d^3}}$$

The equivalent max. shear stress

$$(\tau_{eq})_{max} = \sqrt{(\tfrac{1}{2}\sigma_{eq-n})^2 + (\tau_{eq})^2} = \tau_y/n \rightarrow (\text{eq}^n\ 2.23(a), \text{Pg } 26)$$

$$\frac{205}{1.5} = \sqrt{\left(\frac{10.6298\times10^6}{2d^3}\right)^2 + \left(\frac{3.7298\times10^6}{d^3}\right)^2}$$

$$136.67 = \frac{1}{d^3}\sqrt{(5.3149\times10^6)^2 + (3.7298\times10^6)^2}$$

$$136.67 = \frac{6.4930\times10^6}{d^3}$$

$$\boxed{d = 36.218\,mm}$$

The equivalent max normal stress

$$(\sigma_{eq-n})_{max} = \tfrac{1}{2}\sigma_{eq-n} + \sqrt{(\tfrac{1}{2}\sigma_{eq-n})^2 + (\tau_{eq})^2} = \sigma_y/n \rightarrow [\text{eq}^n\ 2.23(b), \text{Pg } 26]$$

$$(\sigma_{eq-n})_{max} = \frac{10.6298\times10^6}{2d^3} + \sqrt{\left(\frac{10.6298\times10^6}{2d^3}\right)^2 + \left(\frac{3.7298\times10^6}{d^3}\right)^2}$$

$$\frac{410}{1.5} = \frac{5.3149\times10^6}{d^3} + \frac{6.4930\times10^6}{d^3}$$

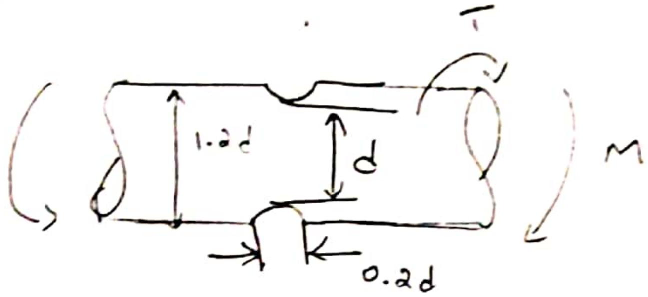
$$273.33 = \frac{11.8079\times10^6}{d^3}$$

$$\therefore \boxed{d = 35.088\,mm}$$

$\therefore$ The minimum diameter of the shaft is 36.218 mm (adopt of bigger value)

5) A round rod of diameter 1.2d has semicircular groove of diameter 0.2d this is to sustain a twisting moment that fluctuates b/w 2.5kN-m & 1.5kN-m. To gether with the bending moment that fluctuates b/w 2kN-m & -1kN-m. Take σ_y = 300MPa, σ_u = 450MPa & FOS 2. find suitable daimeter of the rod.

Soln



Soln

$T_{max} = 2.5kN\text{-}m = 2.5\times10^6 N\text{-}mm$

$T_{min} = 1.5kN\text{-}m = 1.5\times10^6 N\text{-}mm$

$M_{max} = 2kN\text{-}m = 2\times10^6 N\text{-}mm$

$M_{min} = -1kN\text{-}m = -1\times10^6 N\text{-}mm$

$\sigma_y = 300MPa$

$\sigma_u = 450MPa$

FOS = 2.5

$\sigma_{en} = 0.5\times\sigma_u$

$= 0.5\times450$

$\boxed{\sigma_{en} = 225MPa}$

consider a twisting moment

For circular

$$\tau_{max} = \frac{16T_{max}}{\pi d^3} = \frac{16\times2.5\times10^6}{\pi d^3} = \frac{12.7323\times10^6}{d^3}$$

$$\tau_{min} = \frac{16T_{min}}{\pi d^3} = \frac{16\times1.5\times10^6}{\pi d^3} = \frac{7.6394\times10^6}{d^3}$$

$$\tau_{mean} = \frac{\tau_{max}+\tau_{min}}{2} = \frac{12.7323\times10^6+7.6394\times10^6}{2d^3}$$

$$\boxed{\tau_m = \frac{10.18585\times10^6}{d^3}}$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{12.7323\times10^6 - 7.6394\times10^6}{2\,d^3}$$

$$\boxed{\tau_a = \frac{2.54645\times10^6}{d^3}}$$

The equivalent shear stress

$$\tau_{eq} = \tau_m + \left(\frac{\tau_y}{\sigma_{en}}\right)\frac{k_{se}\,\tau_a}{ABC} \rightarrow eq^n\ 22(b),\ Pg\ 2C$$

$$\tau_y = \frac{\sigma_y}{2} = \frac{300}{2}$$

$$\boxed{\tau_y = 150\ MPa}$$

correction factor

$A = 0.5$ (Torsional)

$B = 0.8$

$C = 0.8$

To find the k_{se}

$$k_{se} = 1 + q(k_t - 1)$$

$q = 1$ assume

to find the k_t

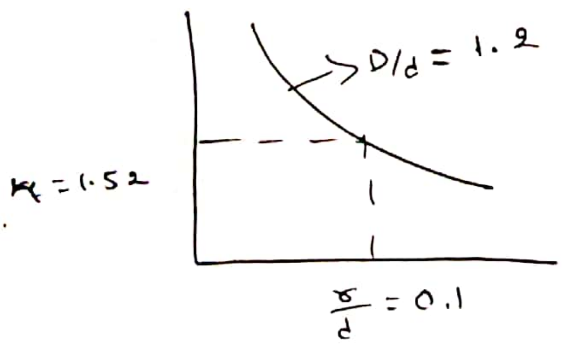
From fig 2.22 Pg 41

$$r/d = \frac{0.1\cancel{d}}{\cancel{d}}$$

$$\boxed{\frac{r}{d} = 0.1}$$

$$\frac{D}{d} = \frac{1.2\cancel{d}}{\cancel{d}}$$

$$\boxed{\frac{D}{d} = 1.2}$$



$$k_{se} = 1 + 1(1.52 - 1)$$

$$\boxed{k_{se} = 1.52}$$

$$\tau_{eq} = \frac{10.1858\times10^6}{d^3} + \left(\frac{150}{225}\right)\frac{1.52\times2.5464\times10^6}{d^3\times0.5\times0.8\times0.8}$$

$$\boxed{\tau_{eq} = \frac{18.2494\times10^6}{d^3}} \rightarrow (1)$$

$S)^n$ considering Bending moment

$$\sigma_{max} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 2 \times 10^6}{\pi d^3} = \frac{20.3718 \times 10^6}{d^3}$$

$$\sigma_{min} = \frac{32 M_{min}}{\pi d^3} = -\frac{32 \times 1 \times 10^6}{\pi d^3} = -\frac{10.1859 \times 10^6}{d^3}$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{20.3718 \times 10^6 - 10.1859 \times 10^6}{2 d^3}$$

$$\boxed{\sigma_m = \frac{5.09295 \times 10^6}{d^3}}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{20.3718 \times 10^6 + 10.1859 \times 10^6}{2 d^3}$$

$$\boxed{\sigma_a = \frac{15.27885 \times 10^6}{d^3}}$$

The equivalent ~~normal~~ normal stress.

$$\sigma_{eq.n} = \sigma_m + \left(\frac{S_y}{\sigma_{en}}\right) \frac{K_{tf}\, \sigma_a}{ABC} \rightarrow eq^n\ 2.22(a)$$

Pg 26.

A = 1 (Bending)

B = 0.8

C = 0.8

To find K_{tf}

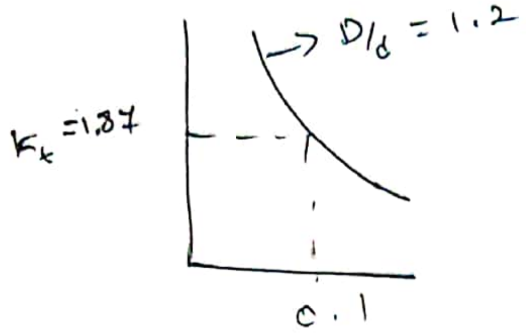
$$K_{tf} = 1 + q(K_t - 1)$$

q = 1 assume

To find K_t

$$\frac{r}{d} = 0.1$$

$$\frac{D}{d} = 1.2$$



$$K_{tf} = 1 + 1(1.87 - 1)$$

$$\boxed{K_{tf} = 1.87}$$

$$\sigma_{eq-n} = \frac{5.09295\times10^{6}}{d^3} + \left(\frac{300}{225}\right)\frac{1.87\times15.2788\times10^{6}}{d^3\times1\times0.8\times0.8}$$

$$\boxed{\sigma_{eq-n} = \frac{64.6166\times10^{6}}{d^3}}$$

The equivalent ~~shear~~ maximum shear stress.

$$\tau_{eq(max)} = \sqrt{\left(\frac{\sigma_{eq-n}}{2}\right)^2 + (\tau_{eq})^2} = \tau_y/n \rightarrow \text{Eq}^n\ 2.23(a)$$
Pg 26

$$\frac{150}{2.5} = \sqrt{\left(\frac{64.6166\times10^{6}}{2d^3}\right)^2 + \left(\frac{18.2494\times10^{6}}{d^3}\right)^2}$$

$$60 = \frac{1}{d^3}\sqrt{(32.3083\times10^{6})^2 + (18.2494\times10}$$

$$60 = \frac{37.1061\times10^{6}}{d^3}$$

$$\boxed{d = 85.1983\text{ mm}}$$

The equivalent max. normal stress

$$\sigma_{eq-n}(max) = \frac{1}{2}\sigma_{eq-n} + \sqrt{\left(\frac{1}{2}\sigma_{eq-n}\right)^2 + (\tau_{eq})^2} = \frac{\sigma_y}{n}$$

$$\frac{300}{2.5} = \frac{64.6166\times10^6}{2d^3} + \sqrt{\left(\frac{64.6166\times10^6}{2d^3}\right)^2 + \left(\frac{18.2494\times10^6}{d^3}\right)}$$

$$120 = \frac{32.3083\times10^6}{d^3} + \frac{37.1061\times10^6}{d^3}$$

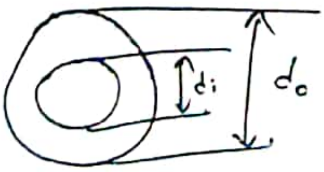
$$120 = \frac{69.4144\times10^6}{d^3}$$

$$\boxed{d = 83.3213\,mm}$$

∴ The minimum diameter of a shaft is 85.1983 mm
(take max value)

6) Determine the diameter of hollow shaft to ~~sustain~~ sustain a twisting movement that fluctuates b/w 2.5 kN-m & 1.5 kN-m together with a bending moment the fluctuates b/w +2 kN-m to -2 kN-m assume the inner diameter to be 0.6 times ~~of~~ outter diameter. Take normal yield stress as 400 MPa & normal endurance stress as 270 MPa & FOS(n) = 2.

soln



$d_i = 0.6 d_o$

$T_{max} = 2.5\times10^6$ N-mm

$T_{min} = 1.5\times10^6$ N-mm

$M_{max} = 2\times10^6$ N-mm

$M_{min} = -2\times10^6$ N-mm

$\sigma_y = 400$ MPa

$\sigma_{en} = 270$ MPa

$n = 2$

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Torsional load

$$\tau = \frac{T \times r}{J} = \frac{16 d_o T}{\pi (d_o^4 - d_i^4)} = \frac{16 d_o T}{\pi (d_o^4 - (0.6 d_o)^4)}$$

$$\tau = \frac{16 d_o T}{\pi d_o^4 (1 - 0.1296)} = \frac{16 \cancel{d_o} T}{0.8704 \pi d_o^4} = \frac{16 T}{0.8704 \pi d_o^3}$$

$$\tau_{max} = \frac{16 T_{max}}{0.8704 \pi d_o^3} = \frac{16 \times 2.5 \times 10^6}{0.8704 \pi d_o^3} = \frac{14.6282 \times 10^6}{d_o^3}$$

$$\tau_{min} = \frac{16 T_{min}}{0.8704 \pi d_o^3} = \frac{16 \times 1.5 \times 10^6}{0.8704 \times \pi d_o^3} = \frac{8.77692 \times 10^6}{d_o^3}$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{(14.6282 \times 10^6) + (8.7769 \times 10^6)}{2 d_o^3}$$

$$\boxed{\tau_m = \frac{11.70255 \times 10^6}{d_o^3}}$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{(14.6282 \times 10^6) - (8.7769 \times 10^6)}{2 d_o^3}$$

$$\boxed{\tau_a = \frac{2.9256 \times 10^6}{d_o^3}}$$

equivivale shear stress

$$\tau_{eq} = \tau_m + \left(\frac{\tau_y}{\sigma_{en}}\right)\left(\frac{K_{sf}\,\tau_a}{ABC}\right) \rightarrow \text{eq}^n\ 2.22(b)\ \text{pg } 26$$

$\tau_y = \frac{\sigma_y}{2} = 200\ \text{MPa}$

assumes, $K_{sf} = 1$

$B = 0.8$

$C = 0.8$

$A = 0.5$ (Torsional)

$$\tau_{eq} = \left(\frac{11.7025 \times 10^6}{d_o^3}\right) + \left(\frac{200}{270}\right)\left(\frac{1 \times 2.9256 \times 10^6}{d_o^3 \times 0.5 \times 0.8 \times 0.8}\right)$$

$$\boxed{\tau_{eq} = \frac{18.4747 \times 10^6}{d_o^3}}$$

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Then load

$\frac{32M}{\pi d}$

$$\sigma_b = \frac{M \times C}{I} = \frac{64 \times d_o/2\ M}{\pi (d_o^4 - d_i^4)} = \frac{32 d_o M}{\pi (d_o^4 - (0.6 d_o)^4)}$$

$$\sigma_b = \frac{32 d_o M}{\pi d_o^4 (0.8704)} = \frac{32 M}{0.8704 \pi d_o^3}$$

$$\sigma_{bmax} = \frac{32 M_{max}}{0.8704 \pi d_o^3} = \frac{32 \times 2 \times 10^6}{0.8704 \times \pi d_o^3} = \frac{23.4051 \times 10^6}{d_o^3}$$

$$\sigma_{bmin} = \frac{32 M_{min}}{0.8704 \pi d_o^3} = \frac{-32 \times 2 \times 10^6}{0.8704 \times \pi d_o^3} = -\frac{23.4051 \times 10^6}{d_o^3}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 0$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{23.405 \times 10^6 + 23.4051 \times 10^6}{2 d^3} = \underline{\frac{23.4051 \times 10^6}{d_o^3}}$$

$$(\sigma_{eq-n}) = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right) \frac{K_f \sigma_a}{ABC}$$

$$= 0 + \left(\frac{400}{270}\right)\left(\frac{1 \times 23.4051 \times 10^6}{1 \times 0.8 \times 0.8\ d_o^3}\right)$$

$$\boxed{\sigma_{eq-n} = \frac{54.1784 \times 10^6}{d_o^3}}$$

$$(\tau_{eq})_{max} = \sqrt{\left(\tfrac{1}{2} \sigma_{eq-n}\right)^2 + (\tau_{eq})^2} = \tau_y/n$$

$$100 = \sqrt{\left(\frac{54.1784 \times 10^6}{2 d_o^3}\right)^2 + \left(\frac{18.4747 \times 10^6}{d_o^3}\right)^2}$$

$$\boxed{d_o = 68.956\ mm}$$

$$(\sigma_{eq-n}) = \frac{1}{2} \sigma_{eq-n} + \sqrt{\left(\tfrac{1}{2} \sigma_{eq-n}\right)^2 + (\tau_{eq})^2} = \sigma_y/n$$

$$\boxed{d_o = 66.89\ mm}$$

$$\boxed{d_o = 68.956\ mm}$$

$d_i = 0.6\, d_o$

$d_i = 0.6 \times 68.956$

$$\boxed{d_i = 41.3736\ mm}$$

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